

TEACHER AND STUDENT RESOURCE

Slopes, lines and proof



NC Math 1 • NC.M1.G-GPE.5

Coordinate geometry • Algebraic reasoning

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.G-GPE.5 | NC Math 1 mathematics

This sample contains 21 complete selected pages from the 108-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76-77

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76-77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems.

- Determine if two lines are parallel, perpendicular, or neither.
- Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

T1 Prove slope criteria, including their assumptions and exceptional cases.

T2 Determine relationships between lines from coordinate evidence.

T3 Construct equations of lines subject to point and slope conditions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: O01, O05, O09, O14, O19, O24, O28

T2: M01, M04, M07, M10, M13, M16, M19, M22, M25, M28, M30, O03, O06, O12, O15, O21, O30

T3: M02, M03, M05, M06, M08, M09, M11, M12, M14, M15, M17, M18, M20, M21, M23, M24, M26, M27, M29, O02, O04, O07, O08, O10, O11, O13, O16, O17, O18, O20, O22, O23, O25, O26, O27, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Prove and explain

Learning target: Prove slope criteria, including their assumptions and exceptional cases.
Select 2–4 items labeled T1, then use one fresh open task as independent evidence.

Session 2: Determine and explain

Learning target: Determine relationships between lines from coordinate evidence.
Select 2–4 items labeled T2, then use one fresh open task as independent evidence.

Session 3: Construct and explain

Learning target: Construct equations of lines subject to point and slope conditions.
Select 2–4 items labeled T3, then use one fresh open task as independent evidence.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Plot ordered pairs, subtract signed numbers, work with exact fractions, and interpret slope as vertical change divided by horizontal change. For polygons, review the Pythagorean theorem and area formulas before using them as evidence.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

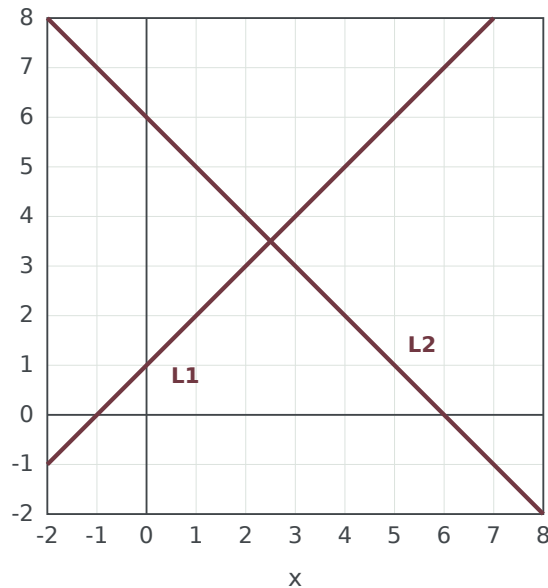
Coordinates are exact. Use exact fractions and radicals unless a task requests an approximation. Axes use equal units. A graph supports an algebraic argument; appearance alone is not proof. Distinct parallel lines never meet; coincident lines are identified separately.

TEACHER MODEL

Model the coordinate reasoning

Find the equation of the line through $P(1, 5)$ that is perpendicular to $y = (1)x + (1)$. Draw and label the relevant lines on the grid. Justify the slope criterion you use and verify the point or intersection conditions.

y



The required slope is -1 . Use point-slope form $y - (5) = (-1)(x - (1))$. Expanding gives $y = (-1)x + (6)$. Substituting $x = 1$ gives $y = 5$; the product $(1)(-1) = -1$. Their intersection is $(5/2, 7/2)$, found by equating the two y expressions. Graph check: the first line passes through $(0, 1)$ and $(1, 2)$; the second passes through $(0, 6)$ and $(1, 5)$. Use these anchors to identify the two plotted lines. In the graph, L1 and L2 identify the first and second plotted lines, respectively.

COORDINATE ANCHOR

Build a verifiable argument

Distance and perimeter

For endpoints (x_1, y_1) , (x_2, y_2) , $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. Take the nonnegative square root for length. Perimeter adds the lengths of all boundary sides once; state linear units.

Area and classification

Rectangle area = base $\times$ perpendicular height; triangle area = half that product. Area uses square units. Equal squared lengths establish equal lengths. Parallel sides need equal slopes or two vertical directions; right angles need perpendicular directions. State sufficient properties for the claimed polygon.

Slope criteria and exceptions

For distinct nonvertical lines, equal slopes characterize parallel lines. If both slopes are finite, perpendicular lines have product -1 . A horizontal and a vertical line are perpendicular. Distinct vertical lines are parallel. Coincident equations describe the same line.

Midpoint and endpoint

$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$. If A and M are known, $B = 2M - A$ coordinate by coordinate. The equal displacements $M - A$ and $B - M$ show both collinearity and equal halves.

A graph is supporting evidence

Plot exact coordinates using the axis labels and equal scales. Use a straightedge. Label vertices, lines and any required midpoint. Join polygon vertices in the stated order. Teacher line diagrams show finite portions of infinite lines. Use algebra to establish conclusions, rather than measuring the printed drawing.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Explain a coordinate proof that two distinct nonvertical lines with equal slopes are parallel. Use equations $y = mx + b$ and $y = mx + c$ with $b \neq c$. Draw one numerical example.

Teacher solution and discussion

If an intersection existed, $mx + b = mx + c$ would imply $b = c$, contradicting distinct intercepts. Therefore the lines never meet and are parallel. For unequal slopes m and n , $(m - n)x = c - b$ has exactly one solution, so distinct nonvertical parallel lines must have equal slopes. Example $y = x + 1$ and $y = x - 2$.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Prove the perpendicular slope criterion for nonvertical lines through the origin using $U(1, m)$, $V(1, n)$ and the converse of the Pythagorean theorem. State any restrictions and show the reverse implication.

Teacher solution and discussion

$OU^2 = 1 + m^2$, $OV^2 = 1 + n^2$ and $UV^2 = (m - n)^2$. A right angle at O is equivalent to $(m - n)^2 = (1 + m^2) + (1 + n^2)$. Expanding and cancelling gives $-2mn = 2$, or $mn = -1$. Conversely $mn = -1$ makes the distance equation true, so the converse Pythagorean theorem gives a right angle. Translating an intersection to the origin preserves angles and slopes. Both slopes must be finite; the vertical/horizontal pair is treated separately.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: O09 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

A direction step for a line is (a, b) , with $a \neq 0$ and $b \neq 0$. Rotate this step 90° to $(-b, a)$. Use coordinates to prove the perpendicular-slope rule and explain why rotation is relevant.

Teacher solution and discussion

The original slope is b/a . A 90° rotation sends the direction to $(-b, a)$, whose slope is $-a/b$. Their product is -1 . Rotation preserves length and makes the two directions perpendicular. Conversely a line of slope $-a/b$ has a direction parallel to $(-b, a)$, hence is perpendicular to the original line. Zero components require the separate horizontal/vertical case.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Prove that the line $x = a$ is perpendicular to the line $y = b$. Explain why multiplying slopes is not valid here. Show an example on the grid.

Teacher solution and discussion

$x = a$ has a vertical direction $(0, 1)$; $y = b$ has a horizontal direction $(1, 0)$. Coordinate axes meet at a right angle, and translating their intersection to (a, b) preserves that angle. The vertical line has undefined slope, so no numerical slope product exists. Example $x = 2$ and $y = -1$ intersect at $(2, -1)$.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Prove that distinct vertical lines are parallel, and explain how equal-slope language must be restricted. Sketch $x = -2$ and $x = 3$.

Teacher solution and discussion

A point on $x = a$ has x-coordinate a . If it were also on $x = c$, then $a = c$. Thus when $a \neq c$ the lines cannot meet and are parallel. Their slopes are undefined, not equal real numbers; the equal-slope criterion applies to nonvertical lines. The example lines have x-coordinates -2 and 3 .

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Use two lines $y = mx + b$ and $y = nx + c$. Derive an intersection formula when $m \neq n$, then use it to prove both directions of the parallel-slope criterion. Sketch an example.

Teacher solution and discussion

Equating y gives $(m - n)x = c - b$, so $x = (c - b)/(m - n)$. Unequal slopes force an intersection. Equal slopes and unequal intercepts give a contradiction $b = c$, so distinct lines do not intersect. Equal slopes and equal intercepts describe the same line, not distinct parallel lines. Example $y = x + 1$ and $y = x - 2$.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Prove slope criteria, including their assumptions and exceptional cases.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Starting with direction vectors $(1, m)$ and $(1, -1/m)$, prove using squared distances that their lines are perpendicular for $m \neq 0$. Explain the missing horizontal case.

Teacher solution and discussion

Their squared lengths are $1 + m^2$ and $1 + 1/m^2$. The squared distance between their endpoints is $(m + 1/m)^2 = m^2 + 2 + 1/m^2$, the sum of the first two squared lengths. The converse Pythagorean theorem gives a right angle at the common origin. If $m = 0$, the perpendicular line is vertical with undefined slope; the reciprocal formula cannot be used.

Evidence and response

Look for correct coordinate differences, an appropriate geometric relationship and a justified conclusion. If a learner relies on appearance, ask which calculation establishes the claim. If coordinate differences are reversed or mixed, use consistent endpoint order for both coordinates, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T2**

Classify the distinct lines $y = (1)x + (-3)$ and $y = (1)x + (-1)$.

- A. Perpendicular
- B. Coincident (the same line)
- C. Neither
- D. Parallel

Reasoning: _____

M02 | Warm-up | T3

Find the equation of the line through $P(-3, -4)$ that is parallel to $y = (-1)x + (-2)$.

- A. $y = (-1)x + (-5)$
- B. $y = (2)x + (-4)$
- C. $y = (1)x + (-7)$
- D. $y = (-1)x + (-7)$

Reasoning: _____

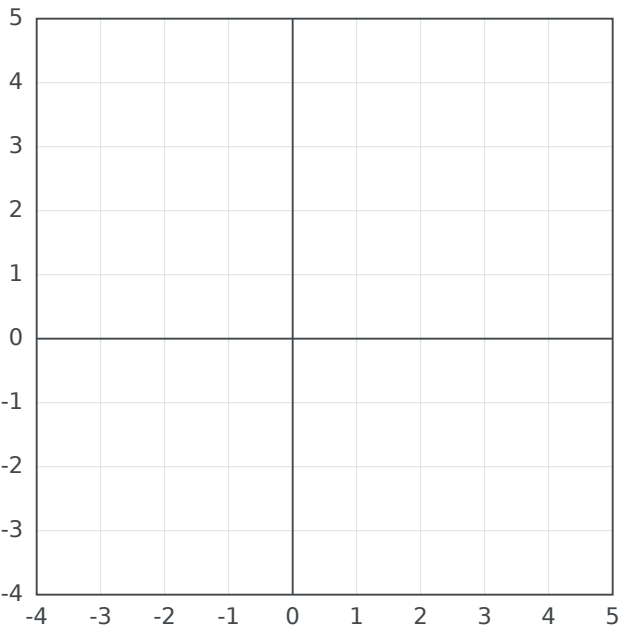
WARM-UP

001 • Show your reasoning

T1: Prove slope criteria, including their assumptions and exceptional cases.

Explain a coordinate proof that two distinct nonvertical lines with equal slopes are parallel. Use equations $y = mx + b$ and $y = mx + c$ with $b \neq c$. Draw one numerical example.

vertical axis: _____



horizontal axis: _____

Explanation and check

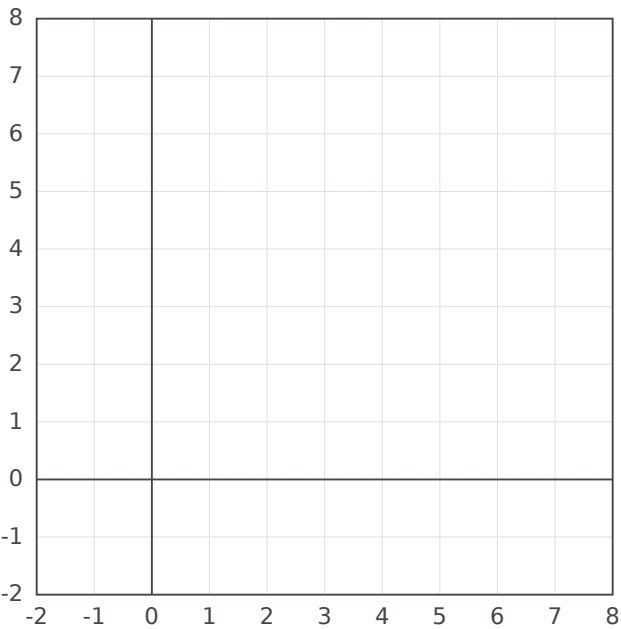
WARM-UP

002 • Show your reasoning

T3: Construct equations of lines subject to point and slope conditions.

Find the equation of the line through P(1, 5) that is perpendicular to $y = (1)x + (1)$. Draw and label the relevant lines on the grid. Justify the slope criterion you use and verify the point or intersection conditions.

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T2: Determine relationships between lines from coordinate evidence.****Answer: D. Parallel**

The slopes are 1 and 1. Their product is 1. The slopes are equal and intercepts differ, so the lines are parallel.

Check each alternative

- A. Does not match the verified result Parallel. Compare the calculation above; a choice alone does not diagnose the learner's reasoning.
- B. Does not match the verified result Parallel. Compare the calculation above; a choice alone does not diagnose the learner's reasoning.
- C. Does not match the verified result Parallel. Compare the calculation above; a choice alone does not diagnose the learner's reasoning.
- D. Matches the verified calculation and all conditions.

M02 • T3: Construct equations of lines subject to point and slope conditions.**Answer: D. $y = (-1)x + (-7)$**

The required slope is -1. Use point-slope form $y - (-4) = (-1)(x - (-3))$. Expanding gives $y = (-1)x + (-7)$. Substituting $x = -3$ gives $y = -4$; the slopes agree.

Check each alternative

- A. Does not match the verified result $y = (-1)x + (-7)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- B. Does not match the verified result $y = (-1)x + (-7)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- C. Does not match the verified result $y = (-1)x + (-7)$. Check the coordinates or substitute the given point; a choice alone does not establish a unique misconception.
- D. Matches the verified calculation and all conditions.

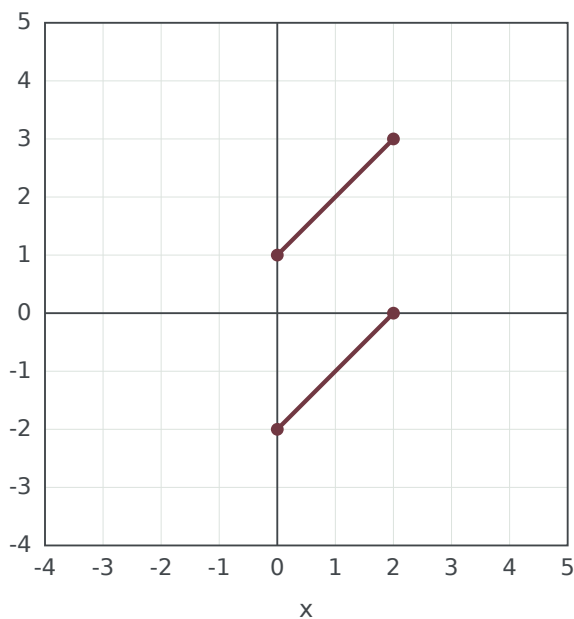
WORKED TEACHER KEYS

001

001 • T1: Prove slope criteria, including their assumptions and exceptional cases.

If an intersection existed, $mx + b = mx + c$ would imply $b = c$, contradicting distinct intercepts. Therefore the lines never meet and are parallel. For unequal slopes m and n , $(m - n)x = c - b$ has exactly one solution, so distinct nonvertical parallel lines must have equal slopes. Example $y = x + 1$ and $y = x - 2$.

y

**Evidence to assess**

A complete proof requires a general argument and its assumptions, not numerical substitution alone. The example is illustrative evidence only.

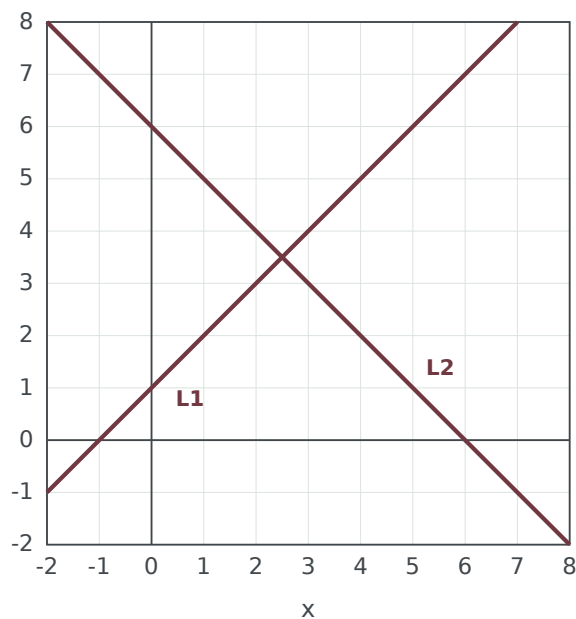
WORKED TEACHER KEYS

002

002 • T3: Construct equations of lines subject to point and slope conditions.

The required slope is -1. Use point-slope form $y - (5) = (-1)(x - (1))$. Expanding gives $y = (-1)x + (6)$. Substituting $x = 1$ gives $y = 5$; the product $(1)(-1) = -1$. Their intersection is $(5/2, 7/2)$, found by equating the two y expressions. Graph check: the first line passes through $(0, 1)$ and $(1, 2)$; the second passes through $(0, 6)$ and $(1, 5)$. Use these anchors to identify the two plotted lines. In the graph, L1 and L2 identify the first and second plotted lines, respectively.

y

**Evidence to assess**

Assess the plotted coordinates, algebraic relationships, logical justification and final conclusion. A correct value without the requested argument is incomplete. Accept equivalent exact forms and other valid proofs.