

TEACHER AND STUDENT RESOURCE

Number Theory and Set Identities



DCS.N.4

Number and Quantity

Seven instructional sections. Thirty multiple-choice and thirty open or performance tasks, original mathematical visuals, generous workspaces, teaching scripts, and worked keys.

Free sample | Contents

DCS.N.4 | NC Discrete Mathematics for Computer Science mathematics

This sample contains 19 complete selected pages from the 92-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand statements related to number theory and set theory.

DCS.N.4.1 Use the Euclidean Algorithm to determine greatest common factor and least common multiple.

DCS.N.4.2 Use the Fundamental Theorem of Arithmetic to solve problems.

DCS.N.4.3 Conclude that sets are equal using the properties of set operations.

DCS.N.4.4 Explain theorems related to greatest common factor, least common multiple, even numbers, odd numbers, prime numbers, and composite numbers.

The decimal codes are the official objectives. All are represented in the item bank. These materials support several sessions, not one 60-task lesson.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

DCS.N.4.1: M01, M02, M03, M04, M19, M20, O01, O02, O03, O04, O05, O06, O07

DCS.N.4.2: M05, M06, M07, M08, M09, M10, O08, O09, O10, O11, O12, O13, O14

DCS.N.4.3: M21, M22, M23, M24, M25, O15, O16, O17, O18, O19, O20

DCS.N.4.4: M11, M12, M13, M14, M15, M16, M17, M18, M26, M27, M28, M29, M30, O21, O22, O23, O24, O25, O26, O27, O28, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Build and interpret representations

Learning target: Use the Euclidean Algorithm to determine greatest common factor and least common multiple. Use the Fundamental Theorem of Arithmetic to solve problems.

Use selected Warm-up and I Do tasks; compare an initial prediction with the worked model.

Session 2 Apply and explain methods

Learning target: Conclude that sets are equal using the properties of set operations. Explain theorems related to greatest common factor, least common multiple, even numbers, odd numbers, prime numbers, and composite numbers.

Use We Do and collaborative tasks across the objectives. Require each learner to explain one mathematical choice.

Session 3 Transfer and collect independent evidence

Learning target: Apply the listed objectives with accurate representations, justified methods, and contextual checks.

Choose independent tasks covering each objective. Reteach an evidenced gap, then use a fresh exit task without answer-supplying cues.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, printed tasks, and optional scratch paper. A calculator may support arithmetic after the method is stated. Use graph paper or movable labels when useful without supplying the requested construction.

Prerequisites

Integer division with remainders, prime factorization, parity, and basic set operations.

Pacing

Select two to four tasks for a session. Preserve first independent evidence, discuss a model, and revisit with a fresh task. The full banks are reusable choices, not a timed requirement.

Neutral access

Read directions aloud without identifying a method or answer. Enlarge diagrams, allow dictated or typed reasoning, and provide accessible tables. Preserve all mathematical conditions.

Keep evidence interpretable

Record neutral supports separately from answer-supplying cues. Retain first responses. A selected distractor is not by itself a diagnosis.

Conventions

gcd means greatest positive common divisor and lcm least positive common multiple. Euclid uses nonnegative remainders smaller than the divisor. A few examples cannot prove a universal statement; one valid counterexample can disprove it.

TEACHER MODEL

Follow the remainder chain

Find $\gcd(84,30)$, then derive $\text{lcm}(84,30)$.

Dividend	Quotient	Divisor	Remainder
84	2	30	24
30	1	24	6
24	4	6	0

Each remainder is nonnegative and smaller than the divisor. The last nonzero remainder is 6. Thus $\gcd=6$ and $\text{lcm}=84 \cdot 30 / 6 = 420$. Check $420 = 5 \cdot 84 = 14 \cdot 30$.

Think aloud: I do not stop at the first remainder. I keep the gcd-preserving pair until the remainder is zero, then use the last nonzero value.

REUSABLE REASONING GUIDE

Represent justify interpret

Givens and conditions

Model and calculation

Justification and independent check

Conclusion in context

WARM-UP

Teaching script

Core 001 | DCS.N.4.1

Collect a short first response before discussion. Ask learners what is given and which condition controls the method.

Display and ask

Use Euclid's algorithm to find $\gcd(252, 198)$. Show every remainder step, and explain why the last nonzero remainder is the gcd.

Work and discuss

$252 = 198 + 54$; $198 = 3 \cdot 54 + 36$; $54 = 36 + 18$; $36 = 2 \cdot 18 + 0$. Thus $\gcd = 18$. At each division, common divisors of the current pair are exactly common divisors of the divisor and remainder, so the gcd is preserved.

Listen for evidence

A complete valid chain and the preservation principle.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M01 for contrast after discussion. Select another task from M01-M04 or O01-O04 to check transfer. Record support and the new evidence separately.

I DO

Teaching script

Core 005 | DCS.N.4.1

Make a prediction before modeling. Think aloud through the core task, making each calculation and check visible.

Display and ask

You have 84 red tiles and 126 blue tiles. Make identical groups using every tile, with the largest possible number of groups. Use Euclid to find the group count and the contents of each group.

Work and discuss

$126 = 84 + 42$; $84 = 2 \cdot 42$, so the greatest group count is $\gcd = 42$. Each group contains $84/42 = 2$ red and $126/42 = 3$ blue tiles. Any allowable group count must divide both totals, so 42 is maximal.

Listen for evidence

Gcd model, division of supplies, and maximality reasoning.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M05 for contrast after discussion. Select another task from M05-M08 or O05-O08 to check transfer. Record support and the new evidence separately.

WE DO

Teaching script

Core 009 | DCS.N.4.2

Ask learners to propose the next step and have a partner verify it against the original conditions.

Display and ask

A learner writes $180=2^2 \cdot 3^2 \cdot 5$. Check the factorization, repair it if needed, and explain how uniqueness of prime factorization can detect inconsistent answers.

Work and discuss

The proposed product is $4 \cdot 9 \cdot 5 = 180$, so it is already correct. There is no repair needed. Every complete prime factorization of 180 must contain exactly two 2s, two 3s, and one 5, apart from order. A different exponent pattern yielding 180 would contradict uniqueness.

Listen for evidence

Check rather than assume an error; state the exact prime multiplicities.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M09 for contrast after discussion. Select another task from M09-M13 or O09-O13 to check transfer. Record support and the new evidence separately.

COLLABORATIVE PRACTICE

Teaching script

Core O14 | DCS.N.4.2

Assign reader, modeler, checker, and explainer roles. Require a private first attempt before comparing approaches.

Display and ask

Explain why a positive integer is a perfect square exactly when all exponents in its prime factorization are even. Address both directions and the case 1.

Work and discuss

If $n=m^2$, squaring m 's prime factorization doubles every exponent, making them even. Conversely, if $n=\prod p_i^{(2e_i)}$, then $n=(\prod p_i^{e_i})^2$. The number $1=1^2$ has an empty prime factorization, so there is no odd exponent to violate the condition.

Listen for evidence

A two-direction proof and the boundary case 1.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M14 for contrast after discussion. Select another task from M14-M18 or O14-O18 to check transfer. Record support and the new evidence separately.

INDEPENDENT PRACTICE

Teaching script

Core O19 | DCS.N.4.3

Provide only neutral access supports. Keep the first response and ask for justification without selecting a method.

Display and ask

Disprove the claim $A \setminus (B \setminus C) = (A \setminus B) \setminus C$. Supply explicit sets, calculate both sides, and explain where the conditions differ.

Work and discuss

Take $A=B=C=\{1\}$. Then $B \setminus C = \emptyset$, so $A \setminus (B \setminus C) = \{1\}$. But $A \setminus B = \emptyset$, so $(A \setminus B) \setminus C = \emptyset$. Difference is not associative: the left side restores B members that also lie in C; the right removes all B members first.

Listen for evidence

A valid counterexample with both parenthesized evaluations.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M19 for contrast after discussion. Select another task from M19-M23 or O19-O23 to check transfer. Record support and the new evidence separately.

SMALL-GROUP RETEACH

Teaching script

Core O24 | DCS.N.4.4

Use the earlier work to identify a specific gap. Model that connection, then remove the cue on a fresh item.

Display and ask

Explain why 2 is the only even prime and why 1 is not an exception to that statement.

Work and discuss

Two has exactly divisors 1 and 2, so it is prime. Any even $n > 2$ is $2(n/2)$ with both factors > 1 , so is composite. One is odd and has only one positive divisor, so it is neither an even number nor a prime.

Listen for evidence

Handle all boundaries with the divisor definition.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M24 for contrast after discussion. Select another task from M24-M27 or O24-O27 to check transfer. Record support and the new evidence separately.

EXIT TICKET

Teaching script

Core O28 | DCS.N.4.4

Collect a concise claim and supporting computation before showing the key. Use the evidence to select a next task.

Display and ask

For positive integers a, b , prove that if $\gcd(a, b) = 1$ then $\text{lcm}(a, b) = ab$. Give a pair of composite relatively prime numbers illustrating the statement.

Work and discuss

The gcd-lcm identity yields $\text{lcm} = ab / \gcd = ab$. Example $a=8, b=15$ are composite and share no prime factor, so $\gcd 1$ and $\text{lcm } 120 = 8 \cdot 15$. Relative primality is a property of a pair, not a requirement that each number be prime.

Listen for evidence

A valid theorem deduction plus an appropriately constrained example.

Probe and respond

Ask which part is an example and which part is a general proof. If a rule is remembered but unsupported, use divisibility or membership to explain why it works before collecting a fresh response.

Next: use M28 for contrast after discussion. Select another task from M28-M30 or O28-O30 to check transfer. Record support and the new evidence separately.

MULTIPLE-CHOICE BANK

M01 / M02 Select and justify**M01 | Warm-up | DCS.N.4.1****Which is $\gcd(84,30)$?**

- A. 6
- B. 12
- C. 24
- D. 420

Reasoning: _____

M02 | Warm-up | DCS.N.4.1**Euclid gives $91=2\cdot 35+21$, $35=21+14$, $21=14+7$, $14=2\cdot 7+0$. Which number is the $\gcd$?**

- A. 14
- B. 21
- C. 0
- D. 7

Reasoning: _____

WARM-UP

001 Show your reasoning

DCS.N.4.1 | Use the Euclidean Algorithm to determine greatest common factor and least common multiple.

Use Euclid's algorithm to find $\gcd(252, 198)$. Show every remainder step, and explain why the last nonzero remainder is the gcd.

Representation and calculation

Explanation and check

WARM-UP

002 Show your reasoning

DCS.N.4.1 | Use the Euclidean Algorithm to determine greatest common factor and least common multiple.

Find $\gcd(105, 84)$ by Euclid, then find the lcm. Verify directly that the lcm is divisible by both original numbers.

Representation and calculation

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 | DCS.N.4.1****Answer A: 6**

Euclid's last nonzero remainder is 6, so $\gcd(84,30)=6$.

- A. Correct. $84=2\cdot 30+24$, $30=24+6$, and $24=4\cdot 6$.
- B. Twelve does not divide 30.
- C. This is an intermediate remainder, not the last nonzero one.
- D. This is the least common multiple, not the gcd.

Evidence: A complete remainder chain with a valid stopping point.

M02 | DCS.N.4.1**Answer D: 7**

The final exact division is $14=2\cdot 7$, so 7 divides the preceding remainder chain and both original numbers.

- A. The process continues past 14 to a smaller nonzero remainder 7.
- B. This is an earlier remainder.
- C. Zero is the stopping remainder, not the greatest positive common divisor.
- D. Correct. It is the last nonzero remainder.

Evidence: Interpret the end of Euclid's algorithm correctly.

WORKED TEACHER KEYS

001 / 002**001 | DCS.N.4.1**

$252=198+54; 198=3\cdot 54+36; 54=36+18; 36=2\cdot 18+0$. Thus $\gcd=18$. At each division, common divisors of the current pair are exactly common divisors of the divisor and remainder, so the gcd is preserved.

Evidence: A complete valid chain and the preservation principle.

002 | DCS.N.4.1

$105=84+21; 84=4\cdot 21$, so $\gcd=21$. $\text{lcm}=105\cdot 84/21=420$. Checks: $420/105=4$ and $420/84=5$. The prime-factor or gcd-lcm argument establishes leastness, not merely these divisibility checks.

Evidence: Gcd chain, lcm derivation, and two quotient checks.