

TEACHER AND STUDENT RESOURCE

Unit-Circle Values, Symmetry and Periodicity



Precalculus • PC.F.2

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

PC.F.2 | North Carolina Precalculus mathematics

This sample contains 21 complete selected pages from the 95-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61, 76-77

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61, 76-77

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of a unit circle with center $(0,0)$ to determine the values of sine, cosine, tangent, cotangent, secant, and cosecant.

T1 PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

T2 PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

PC.F.2.1 Use a unit circle to find values of sine, cosine, and tangent for angles in terms of reference angles.

PC.F.2.2 Explain the relationship between the symmetry of a unit circle and the periodicity of trigonometric functions

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M03, M05, M07, M09, M11, M13, M15, M17, M19, M21, M23, M25, M27, M29, O01, O03, O05, O07, O09, O11, O13, O15, O17, O19, O21, O23, O25, O27, O29

T2: M02, M04, M06, M08, M10, M12, M14, M16, M18, M20, M22, M24, M26, M28, M30, O02, O04, O06, O08, O10, O12, O14, O16, O18, O20, O22, O24, O26, O28, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Learning target: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Coordinate signs, the Pythagorean theorem, special right triangles, radians and fraction operations.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Angles are in radians unless a degree symbol is shown. Positive angles rotate counterclockwise from the positive x-axis; negative angles rotate clockwise. On the unit circle, $P=(\cos\theta, \sin\theta)$. Then $\tan\theta=y/x$, $\cot\theta=x/y$, $\sec\theta=1/\cos\theta$ and $\csc\theta=1/\sin\theta$ whenever the denominator is nonzero. Undefined is not a real number and is never written as infinity. A reference angle is the acute angle to the x-axis for a nonquadrantal terminal ray. A period is a positive input shift preserving every value and the domain; fundamental period means the least positive such shift.

TEACHER REFERENCE

Coordinates first, identities second

Special-angle coordinates

An isosceles right triangle on the unit circle has equal positive legs a with $2a^2=1$, giving $a=\sqrt{2}/2$. Bisecting an equilateral triangle gives the $\pi/6$ and $\pi/3$ coordinates: $(\sqrt{3}/2, 1/2)$ and $(1/2, \sqrt{3}/2)$. Use symmetry rather than memorizing unrelated signs.

Three reflections

Reflection across the x -axis takes (x, y) to $(x, -y)$, corresponding to $-\theta$. Across the y -axis it takes (x, y) to $(-x, y)$, corresponding to $\pi - \theta$. A half-turn takes (x, y) to $(-x, -y)$, corresponding to $\theta + \pi$.

Full and half turns

A full turn 2π returns both coordinates, so all six trigonometric functions repeat where defined. A half-turn changes both coordinate signs; tangent and cotangent ratios remain the same. Sine, cosine, secant and cosecant generally change sign under a half-turn.

Domain matters

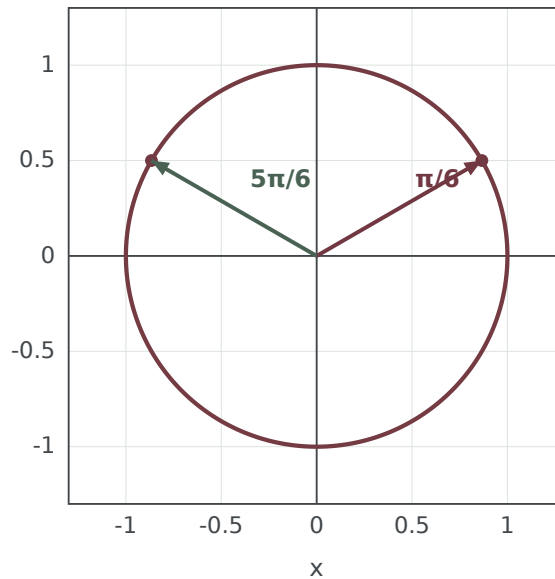
At $(0, \pm 1)$, tangent and secant are undefined. At $(\pm 1, 0)$, cotangent and cosecant are undefined. Reciprocal identities must retain these restrictions. Draw equal scales on both axes so that the unit circle is a circle, not an ellipse.

TEACHER MODEL

A reference triangle gives magnitudes, not all signs

At $\theta = 5\pi/6$, locate the unit-circle point and derive tangent and secant without decimals.

y



The reference angle is $\pi/6$, so coordinate magnitudes are $\sqrt{3}/2$ and $1/2$. The terminal ray lies in quadrant II, giving $P = (-\sqrt{3}/2, 1/2)$. Therefore $\tan\theta = y/x = -\sqrt{3}/3$ and $\sec\theta = 1/x = -2\sqrt{3}/3$. Both are defined because the horizontal coordinate is nonzero. Reflecting the first-quadrant reference point across the y-axis explains the sign change rather than memorizing it.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Use the unit circle and the reference angle to find all six trigonometric values at $\theta = 5\pi/6$. Sketch the terminal point and explain both coordinate signs before taking ratios or reciprocals.

Teacher solution and discussion

The reference angle is $\pi/6$ and the point lies in quadrant II: $P = (-\sqrt{3}/2, 1/2)$. Thus $\sin\theta = 1/2$, $\cos\theta = -\sqrt{3}/2$, $\tan\theta = -\sqrt{3}/3$, $\cot\theta = -\sqrt{3}$, $\sec\theta = -2\sqrt{3}/3$ and $\csc\theta = 2$. The horizontal coordinate is negative and vertical positive; both are nonzero, so all six values are defined.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Reduce $\theta = -17\pi/6$ to a coterminal angle in $[0, 2\pi)$. Give its reference angle, quadrant and exact sine, cosine and tangent. Explain why reducing the angle does not change these values.

Teacher solution and discussion

Add $4\pi = 24\pi/6$ to get $7\pi/6$. Its reference angle is $\pi/6$ and it lies in quadrant III. The point is $(-\sqrt{3}/2, -1/2)$, so sine is $-1/2$, cosine $-\sqrt{3}/2$ and tangent $\sqrt{3}/3$. Adding two full turns returns to the same terminal point, preserving its coordinates and their defined quotient.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Suppose $\sin\theta=5/13$ and θ lies in quadrant II. Determine cosine, tangent, secant, cosecant and cotangent exactly, showing how the quadrant selects a root.

Teacher solution and discussion

$\cos^2\theta=1-25/169=144/169$, so $\cos\theta=\pm 12/13$. Quadrant II requires negative cosine, giving $-12/13$. Then $\tan\theta=-5/12$, $\sec\theta=-13/12$, $\csc\theta=13/5$ and $\cot\theta=-12/5$. The known positive sine and selected negative cosine agree with the stated quadrant.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Prove from unit-circle coordinates that tangent has fundamental period π . Explain both why π works and why no positive shift smaller than π can be a period.

Teacher solution and discussion

A half-turn negates both coordinates and preserves their quotient, so π is a period with matching domain. If a positive $T < \pi$ were a period, $\tan T$ would have to equal $\tan 0 = 0$ and be defined. Tangent is zero only when $y=0$ and $x \neq 0$, at integer multiples of π . There is no such T in $(0, \pi)$, so no smaller positive period exists.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

A learner claims that $\theta=2\pi/3$ has unit-circle point $(1/2, \sqrt{3}/2)$ because its reference angle is $\pi/3$. Identify the mistake, correct the point, and show which of the six proposed positive values need a sign correction.

Teacher solution and discussion

The reference angle gives magnitudes but not quadrant signs. At $2\pi/3$ the point is $(-1/2, \sqrt{3}/2)$, in quadrant II. Sine $\sqrt{3}/2$ and cosecant $2\sqrt{3}/3$ stay positive. Cosine must be $-1/2$, tangent $-\sqrt{3}$, cotangent $-\sqrt{3}/3$ and secant -2 . All functions involving the negative horizontal coordinate have the corresponding sign change.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

Use unit-circle geometry to prove that cosine has fundamental period 2π . Explain why checking only that $\cos 0 = \cos 2\pi$ is not by itself a proof of periodicity.

Teacher solution and discussion

A full turn returns every point, so $\cos(\theta + 2\pi) = \cos \theta$ for all θ . If T is a positive period, then $\cos T = \cos 0 = 1$. The unit-circle point with horizontal coordinate 1 is only $(1, 0)$, reached at $T = 2k\pi$. Thus the least positive candidate is 2π . The one equality at 0 alone checks only one input; periodicity requires equality at every input, supplied by the full-turn argument.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

A student lists $\sin\theta = \sin(\pi - \theta)$ and concludes that π is a period of sine. Explain the logical error with a coordinate argument and an exact example.

Teacher solution and discussion

The identity compares θ with its y-axis reflection $\pi - \theta$, which preserves the vertical coordinate. Their input difference is $\pi - 2\theta$, not a fixed shift independent of θ . A period must be one fixed positive shift that works for every input. At $\theta = \pi/2$, $\sin\theta = 1$ but $\sin(\theta + \pi) = -1$, so π fails. Reflection symmetry is not the same as translation periodicity.

Evidence and response

Ask learners to identify a terminal point, justify its coordinate magnitudes and signs, and only then form a ratio or reciprocal. For symmetry, require a coordinate transformation and its effect on both numerator and denominator. A few matching outputs illustrate an identity but do not prove it for every angle.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****Use a reference angle to find $\sin(5\pi/6)$.**

- A. $-1/2$
- B. $\sqrt{3}/2$
- C. $-\sqrt{3}/2$
- D. $1/2$

Reasoning: _____

M02 | Warm-up | T2**A unit-circle point for θ is (a, b) . What point corresponds to $-\theta$?**

- A. $(a, -b)$
- B. $(-a, b)$
- C. $(-a, -b)$
- D. (b, a)

Reasoning: _____

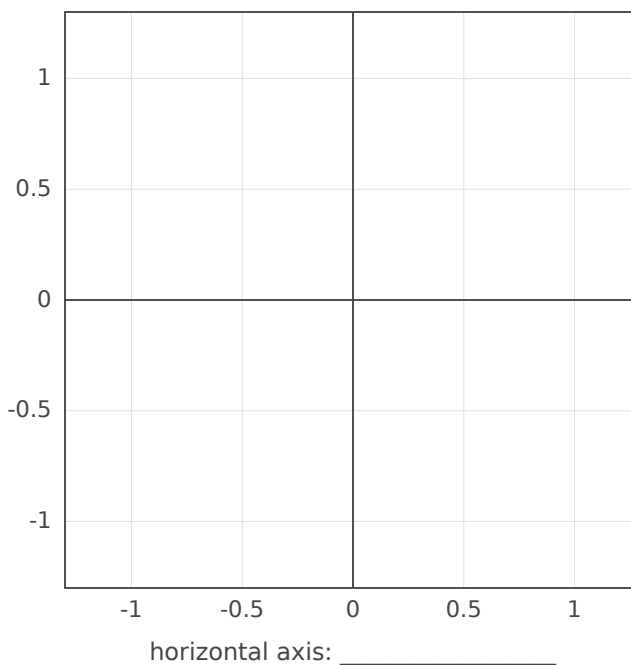
WARM-UP

001 • Show your reasoning

T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Use the unit circle and the reference angle to find all six trigonometric values at $\theta = 5\pi/6$. Sketch the terminal point and explain both coordinate signs before taking ratios or reciprocals.

vertical axis: _____

**Explanation and check**

WARM-UP

002 • Show your reasoning

T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Let $P=(a, b)$ be the unit-circle point for θ . Reflect it across the x-axis, identify the corresponding angle, and derive the even/odd identities for sine, cosine and tangent. State the tangent restriction.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

Answer: D. $1/2$

The reference angle is $\pi/6$. The vertical coordinate is positive in quadrant II, so sine is $1/2$.

Check each alternative

- A. The terminal point is above the x-axis in quadrant II.
- B. That is the magnitude of the horizontal coordinate for this reference angle.
- C. This uses the horizontal magnitude and a negative vertical sign.
- D. Correct. The reference angle is $\pi/6$. The vertical coordinate is positive in quadrant II, so sine is $1/2$.

M02 • T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.

Answer: A. $(a, -b)$

Changing the rotation direction reflects the point across the x-axis: its horizontal coordinate is preserved and its vertical coordinate changes sign.

Check each alternative

- A. Correct. Changing the rotation direction reflects the point across the x-axis: its horizontal coordinate is preserved and its vertical coordinate changes sign.
- B. This reflects across the y-axis, not the x-axis.
- C. This is a half-turn through the origin.
- D. This swaps the coordinates rather than reflecting across the x-axis.

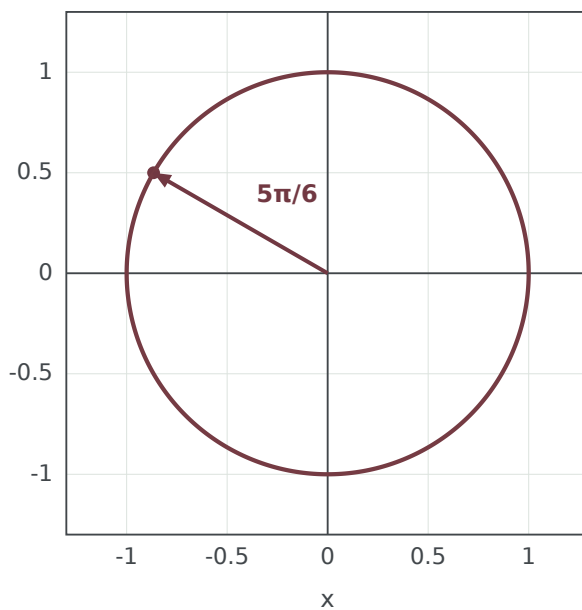
WORKED TEACHER KEYS

001

001 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

The reference angle is $\pi/6$ and the point lies in quadrant II: $P=(-\sqrt{3}/2, 1/2)$. Thus $\sin\theta=1/2$, $\cos\theta=-\sqrt{3}/2$, $\tan\theta=-\sqrt{3}/3$, $\cot\theta=-\sqrt{3}$, $\sec\theta=-2\sqrt{3}/3$ and $\csc\theta=2$. The horizontal coordinate is negative and vertical positive; both are nonzero, so all six values are defined.

y

**Evidence to assess**

Full evidence: The reference angle and point, all six exact values, justified signs and a matching circle/ray sketch. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

WORKED TEACHER KEYS

002 / 003**002 • T2: PC.F.2.2 Explain how unit-circle symmetries establish trigonometric identities and periods.**

The reflection is $(a, -b)$, corresponding to $-\theta$ modulo 2π . Therefore $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$. If $a \neq 0$, $\tan(-\theta) = -b/a = -\tan\theta$. Tangent is undefined on both sides when $a=0$; this is a domain restriction, not an exception with an infinite value. These coordinate relations hold for all terminal points.

Full evidence: Correct reflection and angle, all three identities with $a \neq 0$ for tangent, and a geometric explanation rather than isolated numerical examples. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

003 • T1: PC.F.2.1 Use unit-circle coordinates and reference angles to determine exact trigonometric values.

The reference triangle has equal legs $a > 0$, so $2a^2 = 1$ and $a = \sqrt{2}/2$. The fourth-quadrant point is $(\sqrt{2}/2, -\sqrt{2}/2)$. Hence $\sin\theta = -\sqrt{2}/2$, $\cos\theta = \sqrt{2}/2$, $\tan\theta = -1$, $\cot\theta = -1$, $\sec\theta = \sqrt{2}$ and $\csc\theta = -\sqrt{2}$. Every denominator is nonzero.

Full evidence: An equal-leg Pythagorean derivation, correct fourth-quadrant signs and all six exact values. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.