

TEACHER AND STUDENT RESOURCE

Plane Transformations as Point Functions



NC Math 2 • NC.M2.F-IF.1

Interpreting Functions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.F-IF.1 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Extend the concept of a function to include geometric transformations in the plane by recognizing that:

- the domain and range of a transformation function f are sets of points in the plane;
- the image of a transformation is a function of its pre-image.

T1 Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

T2 Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

T3 Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

T4 Reason about restricted domains, inverse assignments, composed rules and constructions without confusing equal image sets with equal point functions.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04**I Do: M05-M08 and O05-O08****We Do: M09-M13 and O09-O13****Collaborative practice: M14-M18 and O14-O18****Independent practice: M19-M23 and O19-O23****Small-group reteach: M24-M27 and O24-O27****Exit ticket: M28-M30 and O28-O30**

T1: M01, O02, M06, M08, O08, O13, M14, M24, M26, O26

T2: O01, M02, M03, O06, M07, O07, M09, M13, M16, M18, M19, O22, O24, M25, M27, M28

T3: O03, M04, O04, M05, O05, M10, M11, M12, O14, M15, O15, O19, M20, O20, O21, M22, M23, O25, O27, O28, M30

T4: O09, O10, O11, O12, O16, M17, O17, O18, M21, O23, M29, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

Learning target: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Learning target: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Learning target: Reason about restricted domains, inverse assignments, composed rules and constructions without confusing equal image sets with equal point functions.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

Learning target: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Learning target: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Learning target: Reason about restricted domains, inverse assignments, composed rules and constructions without confusing equal image sets with equal point functions.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Coordinate-plane points and ordered pairs, finite sets and membership, basic translations and reflections, simple coordinate rules, segment and filled-region descriptions, and the exactly-one-output function idea.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

A plane point is one object represented by two coordinates. A point transformation is treated here as a function on its stated point-set domain; general plane maps such as projections need not be rigid, one-to-one or shape preserving. The range means the actual image of the stated domain, not a larger set of permitted outputs. Finite sets ignore repetition and listing order. "Filled" includes interior points; a boundary-only or vertex-only domain is explicitly identified. Coordinate diagrams use equal horizontal and vertical unit scales.

TEACHER MODEL

One image point for each pre-image point

Reflect $D = \{(-1, 2), (1, 2), (0, 0)\}$ across the horizontal axis. Describe the domain, every assignment and the range. Explain the roles of the two coordinates and the fixed point.

The rule is $(x, y) \rightarrow (x, -y)$. Its assignments are $(-1, 2) \rightarrow (-1, -2)$, $(1, 2) \rightarrow (1, -2)$, and $(0, 0) \rightarrow (0, 0)$. The domain is the three given points; the range is $\{(-1, -2), (1, -2), (0, 0)\}$. Each ordered pair is one plane point, so each input has one output even though two numbers specify it. The origin is fixed but still has exactly one image. The range describes images actually reached from D , not every point where the formula could have been evaluated.

VISUAL ANCHOR

The function rule follows whole points

Ask what the objects are before counting inputs or testing whether an assignment is a function.

Object or condition	Meaning in the plane
One input	One point such as (2, -3), not two separate numeric inputs.
One image	One point such as (-3, 2), whose two coordinates locate it.
Domain and range	Sets of pre-image points and actually attained image points.
Function requirement	Every domain point receives exactly one image; different inputs may share an image.

A fixed point still has an image. A repeated identical row is not a conflicting assignment. The image of a whole filled figure includes images of its interior, not merely its labeled vertices.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: O01 • T2: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Ask learners to circle whole ordered pairs rather than isolated coordinates. Compare a coordinate projection, a full domain and a range, and verify that the point-image assignments match the stated rule.

Core task

A transformation shifts every point 2 units right and 1 unit down. Its domain is $D = \{(0, 0), (1, 3), (-2, 1)\}$. List each point-image correspondence and the full range. Explain why the domain is a set of three points rather than a set of six numbers.

Teacher solution and discussion

The rule is $(x, y) \rightarrow (x+2, y-1)$. The three correspondences are $(0, 0) \rightarrow (2, -1)$, $(1, 3) \rightarrow (3, 2)$, and $(-2, 1) \rightarrow (0, 0)$. The range is $\{(2, -1), (3, 2), (0, 0)\}$. Each ordered pair is one plane point; its two coordinates are not separate inputs. Thus D has three elements and the range also has three.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T3: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Optional bank: M05–M08; 005–008. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 005, including its visual when present. Model one outgoing arrow for every domain point, including a fixed point. Contrast a vertical input segment with the scalar vertical-line test, and distinguish vertex-only and filled-figure domains.

Core task

Reflect the three-point domain $D = \{(-2, 1), (0, 0), (2, 1)\}$ across the horizontal axis. Draw a complete point-mapping diagram, state the range and explain why the point that stays in place does not break the function rule.

Teacher solution and discussion

The reflection sends (x, y) to $(x, -y)$. Thus $(-2, 1) \rightarrow (-2, -1)$, $(0, 0) \rightarrow (0, 0)$, and $(2, 1) \rightarrow (2, -1)$. The range is $\{(-2, -1), (0, 0), (2, -1)\}$. The origin is a fixed point: it still has exactly one image, even though its image equals its pre-image. Every domain point must have one outgoing arrow, including the fixed point.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T4: Reason about restricted domains, inverse assignments, composed rules and constructions without confusing equal image sets with equal point functions.

Optional bank: M09–M13; 009–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 009, including its visual when present. Use a missing assignment, an ambiguous choice and a deterministic piecewise repair. Compare the forward map with its reversal, and show how restricting the domain can change reverse uniqueness.

Core task

The domain is $D = \{(0, 0), (1, 2), (3, 1)\}$ and the rule is $(x, y) \rightarrow (x+2, y+1)$. Find the range. For each target point $(3, 3)$ and $(5, 5)$, find its possible pre-image under the unrestricted rule and decide whether it is actually reached from D .

Teacher solution and discussion

The range is $\{(2, 1), (3, 3), (5, 2)\}$. Reversing the shift gives pre-image $(u-2, v-1)$. The target $(3, 3)$ has pre-image $(1, 2)$, which belongs to D , so it is reached. The target $(5, 5)$ has pre-image $(3, 4)$, which is not in D , so it is not in this range even though the unrestricted plane translation can reach it.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign domain auditor, image calculator, range collector and function-criterion checker roles. Use composed rules, constant maps, invariant point sets and whole-plane translations to test claims.

Core task

For $D = \{(-1, 1), (1, 1), (1, 3)\}$, first apply $F(x, y) = (x+2, y)$, then apply $P(u, v) = (u, 0)$ to each intermediate image. Make a three-column table, state the final range and explain why the combined rule is a function even though the final range has fewer points than D .

Teacher solution and discussion

The intermediate points are $(1, 1)$, $(3, 1)$, $(3, 3)$; the final images are $(1, 0)$, $(3, 0)$, $(3, 0)$. The final range is $\{(1, 0), (3, 0)\}$. Every original point has one intermediate image and each intermediate point has one final image, so the combined rule assigns one image to each input. A collision of two final images does not make the forward mapping ambiguous.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require complete point-set descriptions, counterexamples and constructions. Distinguish a consistent function from a rule that has not been specified on all the promised inputs.

Core task

Use $T(x, y) = (x^2, y)$ on $D = \{(-2, 1), (2, 1), (0, 1), (2, -1)\}$. Make a complete mapping table, state the range and find all pre-images in D of the target $(4, 1)$. Explain why the map is a function even though the target has more than one pre-image.

Teacher solution and discussion

The images are $(4, 1)$, $(4, 1)$, $(0, 1)$, $(4, -1)$, respectively. The range is $\{(4, 1), (0, 1), (4, -1)\}$. Both $(-2, 1)$ and $(2, 1)$ are pre-images of $(4, 1)$. Every original input still determines exactly one output, so T is a function; unique pre-images are not required by the forward function definition.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Address the observed confusion: scalar coordinates versus points, repeated rows versus conflicting images, allowed outputs versus actual range, or a fixed point versus an unassigned point. Use a fresh whole-point example to check the repair.

Core task

For $D = \{(-2, 1), (0, 3), (2, 1)\}$, reflect across the horizontal axis. A learner writes the domain as $\{-2, 0, 2\}$ and the range as $\{-1, -3\}$. Replace both with the correct point sets and explain exactly what information each scalar list leaves out.

Teacher solution and discussion

The actual domain is $\{(-2, 1), (0, 3), (2, 1)\}$. The reflection gives range $\{(-2, -1), (0, -3), (2, -1)\}$. The proposed domain list keeps only horizontal coordinates, omitting the vertical positions. The proposed range list keeps only distinct vertical coordinates, omitting horizontal positions and merging the two different image points with vertical coordinate -1 .

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T3: Test whether every domain point has exactly one image, distinguishing allowed many-to-one assignments from missing or conflicting images.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Collect a new many-to-one mapping, a boundary-sensitive range, a composition-domain check and an audit of a specified translation. Ask learners to explain what is determined by the pre-image and what still needs a rule.

Core task

Use $T(x, y) = (x+y, 0)$ on $D = \{(-1, 2), (0, 1), (2, 0), (1, 1)\}$. List all images, draw a complete point-mapping diagram and state the range. Explain both why T is a function and why its reversal is not. Address the fact that an image point itself has two coordinates.

Teacher solution and discussion

The images are $(1, 0)$, $(1, 0)$, $(2, 0)$, $(2, 0)$, respectively, so the range is $\{(1, 0), (2, 0)\}$. Every input point determines one image point, making T a function. In reverse, $(1, 0)$ has two possible images and $(2, 0)$ also has two, so the reverse relation is not a function. Each ordered pair such as $(1, 0)$ is one output object, not two separate outputs.

Evidence and response

Assess identification of whole input/output points, completeness of domain and range, exactly-one-image reasoning, correspondence accuracy and justification separately. Do not confuse a many-to-one function with a reverse relation that is not a function. A correct image set alone does not show that every individual assignment follows the rule. Record a domain omission, an ambiguous input and an incorrect coordinate computation as different pieces of evidence.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A plane transformation sends each point (x, y) to $(x+3, y-2)$. What is the image of the single input point $(1, 4)$?

- A. Two separate outputs, 4 and 2
- B. The number 2 only
- C. The point $(4, 2)$
- D. The number 4 only

Reasoning: _____

M02 | Warm-up | T2

A transformation has exactly these assignments: $(1, 2) \rightarrow (3, 2)$ and $(1, -1) \rightarrow (3, -1)$. What is its domain?

- A. $\{1\}$
- B. $\{2, -1\}$
- C. $\{(3, 2), (3, -1)\}$
- D. $\{(1, 2), (1, -1)\}$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T2: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

A transformation shifts every point 2 units right and 1 unit down. Its domain is $D = \{(0, 0), (1, 3), (-2, 1)\}$. List each point-image correspondence and the full range. Explain why the domain is a set of three points rather than a set of six numbers.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

A point map is listed as $(2, 1) \rightarrow (-2, 1)$, $(2, 4) \rightarrow (-2, 4)$, and $(-1, 1) \rightarrow (1, 1)$. A learner says the repeated input coordinate 2 makes the map fail the function rule. Decide whether that claim is valid, list the domain and range, and explain your criterion.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

Answer: C. The point (4, 2)

Substitute both coordinates: $(1+3, 4-2)=(4, 2)$. The input and output are each one point in the plane, represented by an ordered pair.

Check each alternative

- A. The two coordinate numbers together describe one output point.
- B. This gives only the vertical coordinate of the output point.
- C. Substitute both coordinates: $(1+3, 4-2)=(4, 2)$. The input and output are each one point in the plane, represented by an ordered pair.
- D. This gives only the horizontal coordinate of the output point.

Evidence to assess

Require evaluation of both coordinates while treating the image ordered pair as one point.

WORKED TEACHER KEYS

M02

M02 • T2: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

Answer: D. $\{(1, 2), (1, -1)\}$

The domain contains the two input points. They share horizontal coordinate 1 but are distinct plane points because their vertical coordinates differ.

Check each alternative

- A. This records only the distinct horizontal coordinates, not the input points.
- B. This records only vertical coordinates.
- C. This is the range of image points, not the domain.
- D. The domain contains the two input points. They share horizontal coordinate 1 but are distinct plane points because their vertical coordinates differ.

Evidence to assess

Require the full input point set, not either coordinate projection or the image set.

WORKED TEACHER KEYS

001

001 • T2: Identify domain and range as sets of plane points, including every specified vertex, edge, interior point or boundary condition.

The rule is $(x, y) \rightarrow (x+2, y-1)$. The three correspondences are $(0, 0) \rightarrow (2, -1)$, $(1, 3) \rightarrow (3, 2)$, and $(-2, 1) \rightarrow (0, 0)$. The range is $\{(2, -1), (3, 2), (0, 0)\}$. Each ordered pair is one plane point; its two coordinates are not separate inputs. Thus D has three elements and the range also has three.

Pre-image point	Image point
(0, 0)	(2, -1)
(1, 3)	(3, 2)
(-2, 1)	(0, 0)

Evidence to assess

Require all three paired computations, the exact point-set range and an explanation of what counts as one domain element.

WORKED TEACHER KEYS

002

002 • T1: Treat a whole plane point as one input or output, and explain how its image is determined by its pre-image.

The claim is false. The inputs (2, 1) and (2, 4) are different points, not repeated copies of the same input. Each of the three distinct points has exactly one image. The domain is {(2, 1), (2, 4), (-1, 1)} and the range is {(-2, 1), (-2, 4), (1, 1)}. The function criterion applies to whole input points, not to horizontal coordinates alone.

Pre-image	Image
(2, 1)	(-2, 1)
(2, 4)	(-2, 4)
(-1, 1)	(1, 1)

Evidence to assess

Require the two point sets and the exactly-one-image criterion applied to each whole point, explicitly addressing the shared horizontal coordinate.