

TEACHER AND STUDENT RESOURCE

Trigonometry in Models and Triangles



NC Math 4 • NC.M4.AF.2

Advanced Mathematics

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M4.AF.2 | NC Math 4 mathematics

This sample contains 19 complete selected pages from the 94-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75

Using this sample

Read the preparation and access guidance before using a selected activity.

Keep teacher keys and private preparation pages separate from student copies.

Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Apply properties of trigonometry to solve problems.

T1 NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

T2 NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

T3 NC.M4.AF.2.3 Interpret amplitude, period, shifts, midline, domain and range in sinusoidal models.

NC.M4.AF.2.1 Translate trigonometric expressions using the reciprocal and Pythagorean identities.

NC.M4.AF.2.2 Implement the Law of Sines and the Law of Cosines to solve problems.

NC.M4.AF.2.3 Interpret key features (amplitude, period, phase shift, vertical shifts, midline, domain, range) of models using sine and cosine functions in terms of a context.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M04, M07, M10, M13, M16, M19, M22, M25, M28, O01, O04, O07, O10, O13, O16, O19, O22, O25, O28

T2: M02, M05, M08, M11, M14, M17, M20, M23, M26, M29, O02, O05, O08, O11, O14, O17, O20, O23, O26, O29

T3: M03, M06, M09, M12, M15, M18, M21, M24, M27, M30, O03, O06, O09, O12, O15, O18, O21, O24, O27, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Diagnose and model

Learning target: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Learning target: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

Use warm-up evidence to choose explicit I Do models. Collect predictions before revealing the worked solution.

Session 2 Compare and explain

Learning target: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

Learning target: NC.M4.AF.2.3 Interpret amplitude, period, shifts, midline, domain and range in sinusoidal models.

Use selected We Do and collaborative tasks. Ask each learner for an initial representation, then compare methods and checks.

Session 3 Transfer independently

Learning target: Apply the objectives with explicit restrictions, checks and contextual interpretations.

Choose independent items across every objective. Use small-group tasks for a documented gap and a fresh exit task to test transfer.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Right-triangle trigonometry, radians and degrees, algebraic fractions, and reading coordinates.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

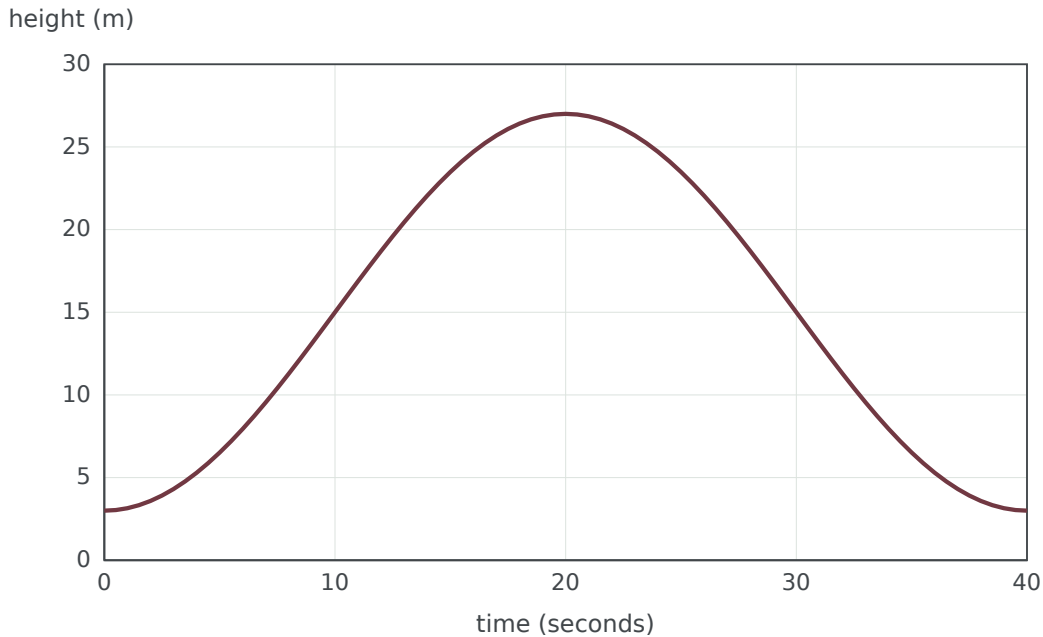
Review convention

Triangle angles use degrees; sinusoidal formula arguments use radians unless marked otherwise. In triangle ABC, side a is opposite A . Round final contextual measurements to two decimals unless requested otherwise. Identities retain the original expression's restrictions. Frequency here means cycles per input unit, the reciprocal of period.

TEACHER MODEL

Read a periodic model in context

A wheel seat follows $h(t)=15-12 \cos(\pi t/20)$ meters for $0 \leq t \leq 40$ seconds. Interpret the model and sketch one cycle.



The midline 15 m is the axle height and amplitude 12 m is the wheel radius. Period = $2\pi/(\pi/20) = 40$ s. The negative cosine starts at the minimum 3 m. Key points are (0, 3), (10, 15), (20, 27), (30, 15), (40, 3). The contextual domain is $[0, 40]$ seconds and range $[3, 27]$ meters.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

Translate $\sec \theta$, $\csc \theta$, and $\cot \theta$ into expressions using sine and cosine. State the denominator restrictions for each.

Teacher solution and discussion

$\sec \theta = 1/\cos \theta$ with $\cos \theta \neq 0$; $\csc \theta = 1/\sin \theta$ with $\sin \theta \neq 0$; $\cot \theta = \cos \theta/\sin \theta$ with $\sin \theta \neq 0$.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

Two straight paths of lengths 8 km and 11 km meet with included angle 40° . Find the distance between their far endpoints. Draw and label the triangle; round to two decimals.

Teacher solution and discussion

The unknown side is opposite the included 40° angle. $c^2 = 8^2 + 11^2 - 2 \cdot 8 \cdot 11 \cos 40^\circ \approx 50.1762$, so $c \approx 7.08$ km. It lies between $|11 - 8| = 3$ and 19, as required.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: NC.M4.AF.2.3 Interpret amplitude, period, shifts, midline, domain and range in sinusoidal models.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

For $y=4 \sin(3(t-2))+1$ with t in seconds, identify amplitude, period, phase shift, and midline. Explain why the shift is 2, not 6.

Teacher solution and discussion

Amplitude 4; period $2\pi/3$ seconds; phase shift 2 seconds right; midline $y=1$. The inner expression is already factored as $3(t-2)$; solving $3(t-2)=0$ places the reference crossing at $t=2$.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Solve the ambiguous case $A=30^\circ$, $a=6$, $b=8$. Find both possible B and C values, then both possible c values to two decimals. Explain why both triangles survive.

Teacher solution and discussion

$\sin B = 8 \sin 30^\circ / 6 = 2/3$. $B_1 \approx 41.81^\circ$, $B_2 \approx 138.19^\circ$. $C_1 \approx 108.19^\circ$, $C_2 \approx 11.81^\circ$. $c = 6 \sin C / \sin 30^\circ = 12 \sin C$ gives $c_1 \approx 11.40$ and $c_2 \approx 2.46$. Each $A+B$ is less than 180° , so both third angles are positive.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

Simplify $(\sec^2\theta - 1)/\sec^2\theta$ using a Pythagorean identity. Keep the original domain and verify with $\theta = \pi/3$.

Teacher solution and discussion

The quotient is $\tan^2\theta/\sec^2\theta = (\sin^2\theta/\cos^2\theta)/(1/\cos^2\theta) = \sin^2\theta$, for $\cos \theta \neq 0$. At $\pi/3$, $(4-1)/4 = 3/4$ and $\sin^2(\pi/3) = 3/4$.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T3: NC.M4.AF.2.3 Interpret amplitude, period, shifts, midline, domain and range in sinusoidal models.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

For $y=2+3 \sin(\pi t/2)$, draw one full cycle from $t=0$ to $t=4$ using five key points. Label the midline, amplitude and period; use a smooth curve rather than line segments.

Teacher solution and discussion

The key points are (0, 2), (1, 5), (2, 2), (3, -1), (4, 2). Midline $y=2$, amplitude 3, period 4. A smooth sinusoidal curve passes through them, rising initially.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Simplify $(1 - \sin^2\theta)/(1 + \tan^2\theta)$. State the restriction and verify with $\theta = \pi/4$.

Teacher solution and discussion

The numerator is $\cos^2\theta$ and denominator is $\sec^2\theta = 1/\cos^2\theta$. The quotient is $\cos^4\theta$, with $\cos \theta \neq 0$. At $\pi/4$, $(1 - 1/2)/(1 + 1) = 1/4$, matching $(\sqrt{2}/2)^4$.

Evidence and response

For identities, require a named identity and denominator restrictions; for triangles, match each side with its opposite angle and check angle sums; for sinusoidal models, connect every parameter to units and the physical situation. Collect a fresh response after any answer-supplying cue.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****For angles where defined, which equals $1/\cos \theta$?**

- A. $\sec \theta$
- B. $\csc \theta$
- C. $\tan \theta$
- D. $\cos^{-1} \theta$

Reasoning: _____

M02 | Warm-up | T2**A triangle has $A=40^\circ$, $B=65^\circ$, and $a=10$. Which equation correctly finds b ?**

- A. $b/\sin 40^\circ = 10/\sin 65^\circ$
- B. $b/\sin 65^\circ = 10/\sin 40^\circ$
- C. $b^2 = 10^2 + 65^2$
- D. $b = 10\cos 65^\circ$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Translate $\sec \theta$, $\csc \theta$, and $\cot \theta$ into expressions using sine and cosine. State the denominator restrictions for each.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

**In triangle ABC, $A=35^\circ$, $B=75^\circ$, and $a=8$ cm. Find C and set up and evaluate
b. Explain why your side order is reasonable.**

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.

Answer: A. $\sec \theta$

The reciprocal identity defines $\sec \theta = 1/\cos \theta$, requiring $\cos \theta \neq 0$.

Check each alternative

- A. Correct. The reciprocal identity defines $\sec \theta = 1/\cos \theta$, requiring $\cos \theta \neq 0$.
- B. Cosecant is $1/\sin \theta$.
- C. Tangent is $\sin \theta/\cos \theta$.
- D. Inverse cosine is an inverse function, not a reciprocal.

M02 • T2: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

Answer: B. $b/\sin 65^\circ = 10/\sin 40^\circ$

The Law of Sines pairs each side with the sine of its opposite angle.

Check each alternative

- A. The known side 10 is opposite 40° , not 65° .
- B. Correct. The Law of Sines pairs each side with the sine of its opposite angle.
- C. Angles cannot be inserted as side lengths.
- D. No right triangle or adjacent/hypotenuse relation is stated.

WORKED TEACHER KEYS

001 / 002**001 • T1: NC.M4.AF.2.1 Rewrite trigonometric expressions using reciprocal and Pythagorean identities.**

$\sec \theta = 1/\cos \theta$ with $\cos \theta \neq 0$; $\csc \theta = 1/\sin \theta$ with $\sin \theta \neq 0$; $\cot \theta = \cos \theta/\sin \theta$ with $\sin \theta \neq 0$.

Full evidence: All three translations and their correct restrictions. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.

002 • T2: NC.M4.AF.2.2 Use the Laws of Sines and Cosines, including possible SSA cases.

$C = 180^\circ - 35^\circ - 75^\circ = 70^\circ$. $b = 8 \sin 75^\circ / \sin 35^\circ \approx 13.47$ cm. Because $B > A$, side b should exceed $a = 8$ cm, which it does.

Full evidence: Angle sum, matched sine ratio, numerical side, and size check. Partial credit: preserve and score a valid intermediate representation or justified step separately from the final accuracy.