

TEACHER AND STUDENT RESOURCE

The Remainder Theorem: Values, Identities and Evidence



NC Math 3 • NC.M3.A-APR.2

Arithmetic with Polynomial and Rational Expressions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M3.A-APR.2 | NC Math 3 mathematics

This sample contains 20 complete selected pages from the 97-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62, 78

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62, 78

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Understand and apply the Remainder Theorem.

T1 Explain and interpret the Remainder Theorem through polynomial identities, divisor conditions and graph values.

T2 Calculate and verify remainders using evaluation, tables, graphs and efficient coefficient records.

T3 Use remainder constraints to infer coefficients, construct polynomials and analyze transformed expressions.

T4 Critique remainder claims by checking domains, exactness, degree restrictions and limits of the available evidence.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M04, M05, M19, M27, M28, O01, O03, O12, O15, O25

T2: M02, M03, M06, M07, M12, M20, M25, M29, O02, O06, O07, O10, O24, O26

T3: M09, M10, M11, M13, M14, M16, M17, M18, M22, O04, O05, O08, O09, O13, O14, O17, O18, O21, O23, O30

T4: M08, M15, M21, M23, M24, M26, M30, O11, O16, O19, O20, O22, O27, O28, O29

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1: Explain and evaluate

Learning target: Derive the theorem from a polynomial identity.

Learning target: Use direct and nested evaluation with signed inputs and missing powers.

Select Warm-up and I Do tasks, comparing coefficient records with substitution. Ask why the divisor vanishes before naming a shortcut.

Session 2: Infer and construct

Learning target: Translate remainder conditions into equations.

Learning target: Construct valid examples and use known values in composed expressions.

Use We Do and Collaborative tasks. For constructions, verify every requested degree, quotient and remainder rather than comparing only with one model answer.

Session 3: Test the scope of a conclusion

Learning target: Separate exact values from approximations and functions from rational representations.

Learning target: Recognize when values do not determine a full polynomial.

Use Independent, targeted Small-group and Exit tasks. Choose a fresh item that tests the identified gap without reusing the same numerical calculation.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Evaluate polynomial expressions at signed and fractional inputs; preserve zero coefficients for missing powers; multiply polynomials and solve simple linear equations. Later tasks extend the linear theorem to determine a quadratic-divisor remainder from two values.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Unless stated otherwise, all coefficients and supplied values are exact real numbers. Divisors are nonzero polynomials. For a linear divisor $ax-b$ with $a \neq 0$, the remainder is the constant $p(b/a)$; a zero remainder is allowed. A polynomial identity is valid at the divisor zero even when its associated rational expression is not. The zero polynomial needs no ordinary degree assignment in these tasks. For construction tasks, accept all examples satisfying every stated requirement.

TEACHER MODEL

The remainder is the value where the divisor vanishes

Divide $p(x)=x^3+2x^2-5x+1$ by $x-2$. Connect evaluation, a coefficient record, a polynomial identity and a point on the graph.

Evaluation gives $p(2)=8+8-10+1=7$. Synthetic division with 2 and coefficients 1,2,-5,1 yields 1,4,3,7. Thus $q=x^2+4x+3$ and $p(x)=(x-2)(x^2+4x+3)+7$. Multiplying back verifies every term. At $x=2$ the product vanishes, so the graph contains (2,7). This is not an x-intercept, and the rational expression $p(x)/(x-2)$ is still undefined at 2.

VISUAL ANCHOR

Four views of one remainder

Divisor $x-c \rightarrow$ solve $x-c=0 \rightarrow$ input c

Division identity $p(x)=(x-c)q(x)+r$

At $x=c$: $p(c)=0 \cdot q(c)+r=r$

Graph $y=p(x)$: the point (c,r) is on the graph

Coefficient record: the last multiply-add value is $p(c)$

Zero remainder: $p(c)=0$, so $p(x)=(x-c)q(x)$ exactly.

The divisor selects the input; the polynomial supplies the output. The output is the remainder, not the quotient and not the input coordinate.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Explain and interpret the Remainder Theorem through polynomial identities, divisor conditions and graph values.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Use O01 to make the sign decision visible: ask which input makes the divisor zero before any arithmetic. On O02, have the learner place a coefficient under every power, including the missing terms. In O03, distinguish a nonzero constant divisor from the zero divisor when the leading parameter disappears. Do not accept a memorized sign rule as the entire explanation.

Core task

A student says that dividing $p(x)=3x^2+2x-5$ by $x+2$ means finding $p(2)$. Explain the sign choice from a division identity, find the actual remainder, and show how the two inputs give different answers.

Teacher solution and discussion

Write $p(x)=(x+2)q(x)+r$. Substituting -2 , not $+2$, makes the divisor factor zero. Thus $r=p(-2)=12-4-5=3$. In contrast $p(2)=12+4-5=11$, the remainder for division by $x-2$. The sign choice comes from solving the divisor equation, not from a memorized instruction to negate every number in a problem.

Evidence and response

Write the identity, solve the divisor-zero equation and distinguish the two independently computed outputs. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: O05 • T3: Use remainder constraints to infer coefficients, construct polynomials and analyze transformed expressions.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Model O05 by translating the remainder into $p(-1)=11$, solving for k , and then checking with a full identity. For O06, run a multiply-add step backward so learners see why the quotient row is not the original coefficient list. In O07 compare exact fraction work with nested multiplication and keep the missing-power position visible.

Core task

Find k so that $p(x)=2x^3+kx^2-3x+4$ leaves remainder 11 on division by $x+1$. Then find the quotient and verify your chosen coefficient in a full division identity.

Teacher solution and discussion

The condition is $p(-1)=-2+k+3+4=k+5=11$, so $k=6$. Synthetic division of coefficients 2,6,-3,4 by $c=-1$ gives 2,4,-7,11. Thus $p(x)=(x+1)(2x^2+4x-7)+11$. Expansion gives $2x^3+6x^2-3x+4$, verifying the coefficient and remainder.

Evidence and response

Use the remainder equation to determine k , then independently reconstruct the complete polynomial. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Use remainder constraints to infer coefficients, construct polynomials and analyze transformed expressions.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

For O09, ask two learners to propose one equation each from the two distinct divisor zeros, then have a third check the solved coefficients in both conditions. Use O10 to expose the nonmonic-divisor issue: solving $2x+1=0$ matters. In O11, contrast the all-input polynomial identity with the restricted rational expression before evaluating.

Core task

Determine a and b if $p(x)=x^3+ax+b$ has remainder 4 on division by $x-1$ and remainder -5 on division by $x+2$. Check both conditions after solving.

Teacher solution and discussion

The first condition gives $p(1)=1+a+b=4$, so $a+b=3$. The second gives $p(-2)=-8-2a+b=-5$, so $-2a+b=3$. Subtracting gives $3a=0$, hence $a=0$ and $b=3$. The polynomial is x^3+3 ; its values are $p(1)=4$ and $p(-2)=-5$ as required.

Evidence and response

Build one equation per divisor zero, solve the system and verify both independent remainder constraints. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T3: Use remainder constraints to infer coefficients, construct polynomials and analyze transformed expressions.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Use roles of constructor, theorem checker and skeptic. In O14, each learner calculates one transformed remainder and another verifies its operations. O16 requires two explicit fitting models, not a verbal assertion that information is insufficient. In O17, track the inner and outer input separately; a known outer value helps only when the inner value reaches it.

Core task

Two unknown polynomials f and g have remainders -2 and 5 when divided by $x+3$. Find the remainders of $f+g$, fg and f^2-2g for the same divisor. Explain why no coefficients of either full polynomial are needed.

Teacher solution and discussion

The supplied data mean $f(-3)=-2$ and $g(-3)=5$. Therefore $(f+g)(-3)=3$, $(fg)(-3)=-10$ and $(f^2-2g)(-3)=4-10=-6$. These values are the three remainders. Polynomial evaluation respects sums and products, so the single needed pair of values determines each requested result without any other coefficients.

Evidence and response

Apply the specified operations to the known values and justify all three conclusions by evaluation. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T4: Critique remainder claims by checking domains, exactness, degree restrictions and limits of the available evidence.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

O19 intentionally asks for a digit-sum conclusion through an unsuitable divisor. Assess whether learners diagnose the mismatch and supply the corrected identity instead of forcing the requested route. O20 asks whether the stated uncertainty interval permits both zero and nonzero remainders; require the interval calculation and both exact examples. In O21 and O22, require the correct remainder degree before applying point-value evidence.

Core task

Represent 234 as $p(10)$, where $p(x)=2x^2+3x+4$. A learner proposes using division by $x-9$ to explain divisibility by 9. Find that polynomial remainder and assess the proposed route. If a different linear divisor gives a more useful digit-sum identity, supply it. Distinguish the relevant polynomial remainder from the integer remainder for 234 divided by 9.

Teacher solution and discussion

For divisor $x-9$, the polynomial remainder is $p(9)=162+27+4=193$. The identity is $p(x)=(x-9)(2x+21)+193$, and at 10 it gives $234=41+193$; this does not directly expose divisibility by 9 because $10-9=1$. The useful divisor is instead $x-1$: $p(1)=9$, and $p(x)=(x-1)(2x+5)+9$. At $x=10$, $234=9\cdot 25+9=9\cdot 26$. The polynomial remainder for $x-1$ is 9, while the integer division remainder for $234\div 9$ is 0 because an integer remainder must lie from 0 through 8. Thus the requested $x-9$ route alone is not the appropriate digit-sum argument; changing to $x-1$ supplies it.

Evidence and response

Identify the mismatch between the polynomial divisor and the intended integer factor, repair it, and distinguish the two kinds of remainder. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T2: Calculate and verify remainders using evaluation, tables, graphs and efficient coefficient records.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Choose O24 for input-output reversal, O25 for lower-degree or zero dividends, O26 for confusing a remainder check with a full quotient check, or O27 for treating equal nonzero values as roots. Preserve the first response. Rebuild the needed identity, then collect evidence on a different item without pointing to the disappearing factor.

Core task

A graph of a polynomial passes through $(-2, -3)$, $(0, 1)$ and $(2, 5)$. State the remainders for division by $x+2$, x and $x-2$. Construct a simple polynomial fitting all three points and sketch it. Explain why the points alone do not prove that this is the only possible polynomial.

Teacher solution and discussion

The remainders are -3 , 1 and 5 , respectively. The linear polynomial $p(x)=2x+1$ fits all three and has a straight-line graph through the given points. But $p(x)=2x+1+x(x+2)(x-2)$ also fits them because the added product vanishes at all three inputs. Therefore the three exact remainders are known, while the full polynomial is not unique without a degree restriction.

Evidence and response

Match each divisor with its graph input, construct a valid graph and explain nonuniqueness rather than infer a global line from three points. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Critique remainder claims by checking domains, exactness, degree restrictions and limits of the available evidence.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

O28 tests whether a learner preserves a rational expression's excluded input while recovering a polynomial. O29 distinguishes a universal rule from an odd-symmetry special case. O30 combines inference and construction. Require an explicit evaluation check; treat a valid alternative construction as full evidence rather than requiring the model example.

Core task

An identity is $p(x)/(x-2)=x^2+1+3/(x-2)$ for $x \neq 2$. Recover p , determine $p(2)$, and explain which part of your reasoning extends to $x=2$ and which displayed expression remains undefined there.

Teacher solution and discussion

For $x \neq 2$, multiplication gives $p(x)=(x-2)(x^2+1)+3=x^3-2x^2+x+1$. Since p is a polynomial and agrees with this polynomial at infinitely many inputs, it is that same polynomial everywhere. Thus $p(2)=8-8+2+1=3$. The polynomial identity is valid at 2, but both rational expressions with denominator $x-2$ remain undefined there; this does not define a value for the quotient.

Evidence and response

Recover the polynomial, justify extending a polynomial identity and keep the original rational expression's excluded input. Ask the learner to point to the calculation or representation supporting the claim. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

To find the remainder when a polynomial $p(x)$ is divided by $x+3$, which input should be substituted into p ?

- A. 3.
- B. $1/3$.
- C. 0.
- D. -3.

Reasoning: _____

M02 | Warm-up | T2

What is the remainder when $p(x)=2x^2-5x+4$ is divided by $x-2$?

- A. 4.
- B. 2.
- C. 0.
- D. 22.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Explain and interpret the Remainder Theorem through polynomial identities, divisor conditions and graph values.

A student says that dividing $p(x)=3x^2+2x-5$ by $x+2$ means finding $p(2)$. Explain the sign choice from a division identity, find the actual remainder, and show how the two inputs give different answers.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T2: Calculate and verify remainders using evaluation, tables, graphs and efficient coefficient records.

Divide $2x^4 - x^2 + 3$ by $x - 2$. Show a coefficient record that preserves missing powers, find both quotient and remainder, and multiply back to check the complete identity.

Power in dividend	Coefficient, including zeros	Multiply-add record

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01 / M02

M01 • T1: Explain and interpret the Remainder Theorem through polynomial identities, divisor conditions and graph values.

Answer: D. -3.

Write $x+3$ as $x-(-3)$. In $p(x)=(x+3)q(x)+r$, substitution $x=-3$ makes the product zero, leaving $p(-3)=r$.

Check each alternative

- A. The divisor becomes zero at -3, not at 3.
- B. Taking a reciprocal does not solve $x+3=0$.
- C. Zero applies to division by x , not $x+3$.
- D. Correct. Write $x+3$ as $x-(-3)$. In $p(x)=(x+3)q(x)+r$, substitution $x=-3$ makes the product zero, leaving $p(-3)=r$.

Evidence to assess

Identify the zero of the divisor and explain why that input isolates the remainder.

M02 • T2: Calculate and verify remainders using evaluation, tables, graphs and efficient coefficient records.

Answer: B. 2.

The divisor is $x-2$, so $r=p(2)=2(2^2)-5(2)+4=8-10+4=2$. A full quotient calculation is not needed to determine this constant remainder.

Check each alternative

- A. The constant term is the remainder only when the divisor is x .
- B. Correct. The divisor is $x-2$, so $r=p(2)=2(2^2)-5(2)+4=8-10+4=2$. A full quotient calculation is not needed to determine this constant remainder.
- C. Substitution gives $8-10+4=2$, not zero.
- D. This is $p(-2)$, but $x-2$ vanishes at $+2$.

Evidence to assess

Use the divisor zero and carry out the signed substitution accurately.

WORKED TEACHER KEYS

001 / 002

001 • T1: Explain and interpret the Remainder Theorem through polynomial identities, divisor conditions and graph values.

Write $p(x)=(x+2)q(x)+r$. Substituting -2 , not $+2$, makes the divisor factor zero. Thus $r=p(-2)=12-4-5=3$. In contrast $p(2)=12+4-5=11$, the remainder for division by $x-2$. The sign choice comes from solving the divisor equation, not from a memorized instruction to negate every number in a problem.

Evidence to assess

Write the identity, solve the divisor-zero equation and distinguish the two independently computed outputs.

002 • T2: Calculate and verify remainders using evaluation, tables, graphs and efficient coefficient records.

The full coefficient list is $2, 0, -1, 0, 3$. Synthetic division with 2 gives running values $2, 4, 7, 14, 31$. Hence $q=2x^3+4x^2+7x+14$ and $r=31$. Multiplying $(x-2)q$ gives $2x^4-x^2-28$, and adding 31 gives $2x^4-x^2+3$. Also $p(2)=32-4+3=31$, checking the remainder independently.

Evidence to assess

Preserve zero coefficients, interpret the quotient degrees correctly and verify the full reconstruction, not only the final entry.