

TEACHER AND STUDENT RESOURCE

Every Quadratic Solution and the Formula Behind It



NC Math 2 • NC.M2.A-REI.4

Reasoning with Equations and Inequalities

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.A-REI.4 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 123-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 31-32, 61-62, 91-92

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 31

O02: sample PDF page(s) 19; original page(s) 32

Selected teacher prompts and keys: original page(s) 61-62, 91-92

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Solve for all solutions of quadratic equations in one variable.

- a. Understand that the quadratic formula is the generalization of solving $ax^2 + bx + c$ by using the process of completing the square.
- b. Explain when quadratic equations will have non-real solutions and express complex solutions as $a \pm bi$ for real numbers a and b .

T1 Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

T2 Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

T3 Explain when a quadratic has distinct real, repeated real or non-real roots using discriminants, completed forms and parameter conditions. (4b)

T4 Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, O01, M03, O03, M14, O14

T2: M05, O05, M06, O06, M07, O07, M11, O11, M19, O19, M28, O28

T3: M04, O04, M09, O09, M12, O12, M13, O13, M17, O17, M20, O20, M22, O22, M26, M29, O29

T4: M02, O02, M08, O08, M10, O10, M15, O15, M16, O16, M18, O18, M21, O21, M23, O23, M24, O24, M25, O25, O26, M27, O27, M30, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Learning target: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Learning target: Explain when a quadratic has distinct real, repeated real or non-real roots using discriminants, completed forms and parameter conditions. (4b)

Learning target: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Learning target: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Learning target: Explain when a quadratic has distinct real, repeated real or non-real roots using discriminants, completed forms and parameter conditions. (4b)

Learning target: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Factoring, completing the square, signed coefficients, radical simplification, $i^2 = -1$, complex-number addition and multiplication, and real-coordinate quadratic graphs.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

Solve over the complex numbers unless a task explicitly asks for real or contextual solutions. Coefficients and parameters are real unless stated otherwise. A genuine quadratic has a nonzero leading coefficient. For a positive real r , the principal value $\sqrt{-r} = i\sqrt{r}$ is one number; solving $z^2 = -r$ requires both $\pm i\sqrt{r}$. In $a \pm bi$ form, a and b are real. A repeated root is one distinct value with multiplicity two. The quoted standard's part 4a omits an equals sign in its source; instructional equations here explicitly use $=0$.

TEACHER MODEL

A completed square and the formula tell the same story

Solve $4x^2-12x+13=0$ in two ways and explain why the answers must agree.

Complete: $4(x-3/2)^2+4=0$, so $(x-3/2)^2=-1$ and $x=3/2\pm i$. With $a=4$, $b=-12$ and $c=13$, the discriminant is $144-208=-64$. The formula gives $x=(12\pm 8i)/8=3/2\pm i$. Both methods use the same perfect-square structure; the general formula packages that completion for arbitrary coefficients. Check either root: $4(\pm i)^2+4=-4+4=0$. The negative discriminant explains the non-real pair, not an absence of complex solutions.

VISUAL ANCHOR

The discriminant records the completed-square outcome

For real a , b and c with $a \neq 0$, completing the square leads to $(2ax+b)^2=D$, where $D=b^2-4ac$. Solving then gives $x=(-b \pm \sqrt{D})/(2a)$.

Discriminant	Root type	Example and complete roots
$D > 0$	Two distinct real roots	$x^2+2x-8=0$: $x=2$ or -4
$D = 0$	One repeated real root	$x^2+2x+1=0$: $x=-1$ twice
$D < 0$	Two non-real conjugate roots	$x^2+2x+10=0$: $x=-1 \pm 3i$

The sign of a alone does not decide whether roots are real. A real graph shows only real inputs: no x -intercepts can coexist with two non-real complex solutions. Always retain both branches, even when they later coincide.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Check signed coefficients, zero-product reasoning and $i^2 = -1$ before introducing the formula. Ask for every root and distinguish distinct roots from multiplicity.

Core task

Solve $x^2 - x - 12 = 0$. Choose a suitable method, explain why it fits the equation, and check every solution in the original polynomial.

Teacher solution and discussion

The integers -4 and 3 multiply to -12 and add to -1, so factoring is efficient: $(x-4)(x+3)=0$. Thus $x=4$ or -3. Checks: $16-4-12=0$ and $9+3-12=0$. Either zero factor makes the product zero; neither root may be omitted.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T2: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Derive the formula from a completed square instead of presenting it as an unexplained rule. Mark $a \neq 0$, the added compensation, both square-root branches and the scope of the denominator.

Core task

Starting with $x^2 + bx + c = 0$ for real b and c , derive the monic quadratic formula by completing the square. Explain how a negative discriminant changes the type of solution, not the validity of the derivation.

Teacher solution and discussion

Move c : $x^2 + bx = -c$. Add $b^2/4$ to both sides: $(x + b/2)^2 = b^2/4 - c = (b^2 - 4c)/4$. Let $D = b^2 - 4c$. Taking both square-root branches gives $x + b/2 = \pm \sqrt{D}/2$, hence $x = (-b \pm \sqrt{D})/2$. If $D < 0$, write $\sqrt{D} = i\sqrt{(-D)}$; the same algebra yields the non-real pair $-b/2 \pm [\sqrt{(-D)}/2]i$. If $D = 0$ the branches coincide, and if $D > 0$ they are distinct real values.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Explain when a quadratic has distinct real, repeated real or non-real roots using discriminants, completed forms and parameter conditions. (4b)

Optional bank: M09–M13; 009–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page 009, including its visual when present. Compare negative, zero and positive discriminants with completed forms. Include a parameter value that makes $a=0$ and ask why that member must be treated separately.

Core task

For the real parameter k , classify the roots of $x^2-6x+k=0$ as k varies. Then solve the cases $k=5$, $k=9$ and $k=13$, and relate each result to the completed form.

Teacher solution and discussion

Complete: $(x-3)^2=9-k$. Equivalently $D=36-4k=4(9-k)$. If $k<9$ there are two distinct real roots; if $k=9$ there is a repeated real root 3; if $k>9$ there are two non-real roots $3\pm i\sqrt{k-9}$. At $k=5$, $x=3\pm 2$, giving 1 and 5. At $k=9$, $x=3$. At $k=13$, $x=3\pm 2i$. The sign of $9-k$ explains all cases directly.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T1: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign solver, branch checker, algebra verifier and counterexample constructor roles. Compare methods, verify complex claims, construct coefficients and track how graph changes affect root type.

Core task

Solve $5(x-3)^2 = -20$ by the isolated-square method, then confirm the same root set with the quadratic formula applied to the expanded equation. Recommend a method for this form without claiming the other is invalid.

Teacher solution and discussion

Divide by 5: $(x-3)^2 = -4$, so $x = 3 \pm 2i$. Expansion gives $5x^2 - 30x + 65 = 0$. Its discriminant is $900 - 1300 = -400$, so the formula gives $(30 \pm 20i)/10 = 3 \pm 2i$. The isolated-square method is shorter because it uses the visible structure; the formula is equally valid. Substitution gives $5(\pm 2i)^2 = 5(-4) = -20$ for both roots.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require complete exact solutions and explanations for unfamiliar forms. Include impossibility arguments, contextual interpretation, fractional coefficients and the relationship between real coefficients and conjugate roots.

Core task

A derivation reaches $(x+b/(2a))^2=D/(4a^2)$, where $a \neq 0$ and $D=b^2-4ac$. Explain why the principal square root of $4a^2$ is $2|a|$, not always $2a$. Then show why the complete $\pm$ solution set still simplifies to the standard quadratic formula even when $a < 0$.

Teacher solution and discussion

For real a , $\sqrt{4a^2}=2|a|$, which is nonnegative. Taking both branches gives $x=-b/(2a) \pm \sqrt{D}/(2|a|)$, with the usual complex interpretation if $D < 0$. If $a > 0$, $|a|=a$ directly. If $a < 0$, $|a|=-a$, so the radical term has the opposite sign from $\sqrt{D}/(2a)$; the $\pm$ pair includes both signs anyway. Relabeling the two branches gives the same set $x=(-b \pm \sqrt{D})/(2a)$. The individual plus/minus labels may swap, but no root is lost.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use the observed gap: omitted imaginary branch, missing a in $2a$, incorrect i^2 , incomplete division of a numerator or confusion between a principal root and an equation's full solution set.

Core task

A learner evaluates $\sqrt{-32}$ as $\pm 4\sqrt{2}i$, then uses only $x = -3 + 4\sqrt{2}i$ to solve $(x+3)^2 = -32$. Repair both notation choices and give a check of every equation solution.

Teacher solution and discussion

The principal value is $\sqrt{-32} = 4\sqrt{2}i$, with no $\pm$ in that single evaluation. But solving the square equation requires $x+3 = \pm 4\sqrt{2}i$, so $x = -3 \pm 4\sqrt{2}i$. Either solution gives $(x+3)^2 = (\pm 4\sqrt{2}i)^2 = 32i^2 = -32$. The learner used $\pm$ where one principal value was requested and then omitted it where both equation branches were needed.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T2: Connect completing the square to the general quadratic formula, including the nonzero leading-coefficient condition. (4a)

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Ask for symbolic transfer, parameter boundaries and an exact complex-root check. Do not count a memorized formula alone as evidence that the learner understands its completing-square origin.

Core task

For real p and positive real q , solve $x^2 - 2px + (p^2 + q^2) = 0$ in terms of p and q . Derive the same result by completing the square and by the quadratic formula. Explain what changes if $q = 0$ is permitted.

Teacher solution and discussion

Completion gives $(x - p)^2 + q^2 = 0$, so $(x - p)^2 = -q^2$ and $x = p \pm qi$ because $q > 0$. The discriminant is $(-2p)^2 - 4(p^2 + q^2) = -4q^2$, giving $x = (2p \pm 2qi)/2 = p \pm qi$. Either root makes $(x - p)^2 + q^2 = (\pm qi)^2 + q^2 = 0$. If $q = 0$, both branches coincide at the repeated real root p instead of two distinct non-real roots.

Evidence and response

Assess method choice, equivalence steps, completion compensation, signed coefficient substitution, complete branches and exact verification separately. Distinguish no real roots from no complex roots, a real graph from a complex root set, and one principal square-root value from both solutions of a square equation. Parameter boundaries and degenerate nonquadratic cases require explicit attention.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1****What is the complete solution set of $x^2-5x+6=0$?**

- A. {2, 3}
- B. {2}
- C. {5, 6}
- D. {-2, -3}

Reasoning: _____

M02 | Warm-up | T4**Over the complex numbers, what are all solutions of $x^2=-16$?**

- A. $x=4$ only
- B. $x=\pm 4i$
- C. $x=\pm 16i$
- D. $x=\pm 4$

Reasoning: _____

WARM-UP

001 • Show your reasoning

T1: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Solve $x^2 - x - 12 = 0$. Choose a suitable method, explain why it fits the equation, and check every solution in the original polynomial.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T4: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Solve $(x-2)^2 = -9$ over the complex numbers. Express both answers as $a+bi$ and verify them using $i^2 = -1$.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

Answer: A. {2, 3}

Factor as $(x-2)(x-3)=0$. Either factor can vanish, giving $x=2$ or 3. Checks: $4-10+6=0$ and $9-15+6=0$.

Check each alternative

A. Factor as $(x-2)(x-3)=0$. Either factor can vanish, giving $x=2$ or 3. Checks: $4-10+6=0$ and $9-15+6=0$.

B. This omits the second zero factor and its solution 3.

C. The coefficients are not automatically the roots.

D. These signs would solve the factors $x+2$ and $x+3$, not the displayed equation.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

M02

M02 • T4: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

Answer: B. $x = \pm 4i$

Since $i^2 = -1$, $(4i)^2 = (-4i)^2 = -16$. Both signs are required, so the solutions are $4i$ and $-4i$.

Check each alternative

- A. A real 4 squares to positive 16 and also omits a branch.
- B. Since $i^2 = -1$, $(4i)^2 = (-4i)^2 = -16$. Both signs are required, so the solutions are $4i$ and $-4i$.
- C. These values square to -256, not -16.
- D. Both real values square to positive 16.

Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

WORKED TEACHER KEYS

001

001 • T1: Solve for all quadratic solutions using a method suited to the equation and verify the complete root set.

The integers -4 and 3 multiply to -12 and add to -1, so factoring is efficient: $(x-4)(x+3)=0$. Thus $x=4$ or -3 . Checks: $16-4-12=0$ and $9+3-12=0$. Either zero factor makes the product zero; neither root may be omitted.

Evidence to assess

Require a justified method, equivalent factorization, both roots and explicit original checks.

WORKED TEACHER KEYS

002

002 • T4: Express, construct and verify non-real quadratic solutions as $a \pm bi$ for real a and b , retaining both conjugate branches. (4b)

The values whose square is -9 are $\pm 3i$. Thus $x-2 = \pm 3i$ and $x = 2 + 3i$ or $2 - 3i$. For either answer, $(x-2)^2 = (\pm 3i)^2 = 9i^2 = -9$. The shared real part is 2 and the imaginary coefficients are 3 and -3 .

Evidence to assess

Require both standard-form complex answers, translation by 2 , and a square check valid for both signs.