

TEACHER AND STUDENT RESOURCE

Equivalent Forms Reveal Features



NC Math 1 • NC.M1.F-IF.8

Interpreting Functions • Objectives a and b

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M1.F-IF.8 | NC Math 1 mathematics

This sample contains 20 complete selected pages from the 95-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-15, 30-31, 60, 75-76

Representative activities

M01: sample PDF page(s) 16; original page(s) 15

M02: sample PDF page(s) 16; original page(s) 15

O01: sample PDF page(s) 17; original page(s) 30

O02: sample PDF page(s) 18; original page(s) 31

Selected teacher prompts and keys: original page(s) 60, 75-76

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use equivalent expressions to reveal and explain different properties of a function.

- a. Rewrite a quadratic function to reveal and explain different key features of the function
- b. Interpret and explain growth and decay rates for an exponential function.

T1 Rewrite quadratics to reveal and explain zeros and intercepts (8a).

T2 Use equivalent quadratic forms to explain vertices, extrema, and range (8a).

T3 Interpret exponential initial values, multipliers, and growth or decay rates (8b).

T4 Rewrite exponential expressions to explain rates over different input units (8b).

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M04, M05, M06, M13, M15, O01, O02, O03, O04, O05, O06, O11, O13

T2: M07, M08, M09, M10, M11, M12, M14, M16, O07, O08, O09, O10, O12, O14

T3: M17, M18, M19, M20, M21, M22, M25, M27, M29, O15, O16, O17, O18, O19, O20, O25, O27, O29

T4: M23, M24, M26, M28, M30, O21, O22, O23, O24, O26, O28, O30

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Expand and factor quadratics, use zero products and nonnegative squares, calculate with rational exponents and percentages, and evaluate functions.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

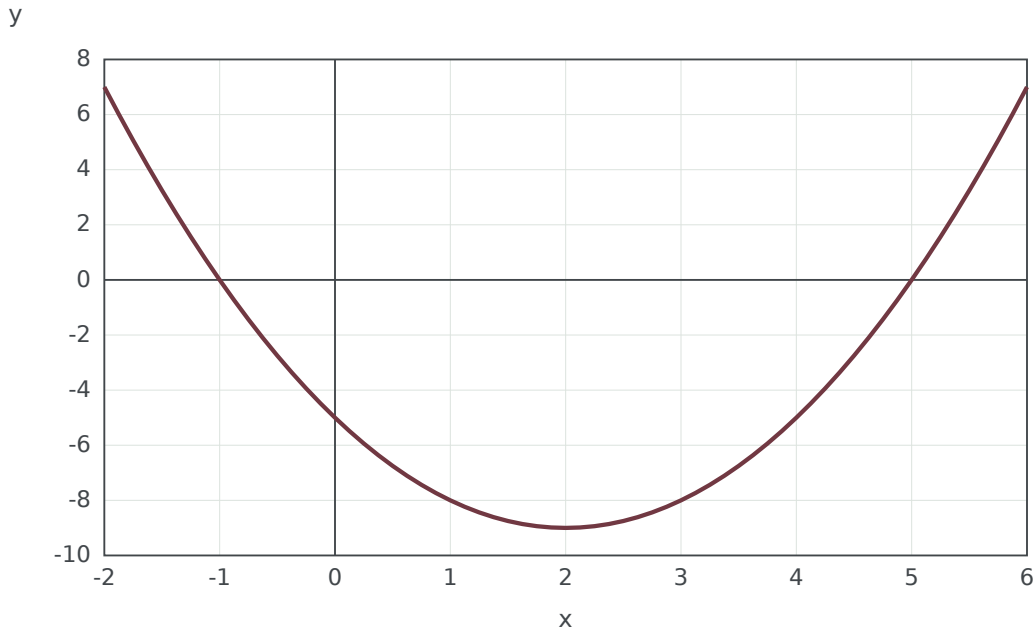
Review convention

Keep exact fractions or terminating decimals. Functions have real domains unless a context restricts them. For exponential models, identify the time unit before interpreting the base. Graph curves continue beyond the printed window; no plotted boundary is a domain endpoint.

TEACHER MODEL

One quadratic, three useful views

Rewrite $f(x)=x^2-4x-5$ to explain its zeros, extremum and y-intercept.



Factor: $(x-5)(x+1)$, revealing zeros 5 and -1 . Complete the square: $(x-2)^2-9$, revealing minimum -9 at $x=2$. Expanded constant -5 is $f(0)$, the y-intercept. Expand each rewrite to verify equality. A table at $x=0,1,2,3,4$ gives $-5,-8,-9,-8,-5$; symmetric outputs agree with axis $x=2$. Say: “An equivalent expression changes what is visible, not the function’s values.”

VISUAL ANCHOR

Choose a form for the feature

Rewriting must preserve every output on the original domain.

Form	Visible feature	Reason
$a(x-r)(x-s)$	Zeros r and s	Zero-product property
$a(x-h)^2+k$	Vertex (h,k)	A square is nonnegative
ax^2+bx+c	y -intercept $(0,c)$	Set $x=0$
$A(1\pm r)^t$	Initial value and rate	A at $t=0$; multiplier each step

A multiplier 1.15 means 15% growth. A multiplier .85 means 15% decay. A time-unit change may require a power or root of the base.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Givens, quantities and restrictions**Representation and calculation****Justification or check****Conclusion, domain and units**

WARM-UP

Teaching script

Core: 001 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its graph when present. Invite a first independent response before discussion. Ask, “What do the symbols represent? What evidence supports your claim?”

Core task

For $f(x) = x^2 + 2 \cdot x - 8$, rewrite in factored and vertex forms. Explain which form reveals the zeros, axis, extremum, and y-intercept most directly. Make a five-point table including the vertex and graph the function on the printed window.

Teacher solution and discussion

Factored: $1(x - (-4))(x - (2))$. Zeros: -4, 2. Vertex: $1(x - (-1))^2 + (-9)$. Axis $x = -1$; minimum -9.

Expanded constant gives $f(0) = -8$. Example table at $x = h - 2, h - 1, h, h + 1, h + 2$: outputs -5, -8, -9, -8, -5. Plot these symmetric points and a smooth parabola. Expanding either rewrite gives $x^2 + 2 \cdot x - 8$.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its graph when present. Think aloud through the core task. Say, “I will identify the quantities first, then build a representation and check it against the stated domain.” Hide the worked key until after predictions.

Core task

For $f(x)=3\cdot x^2 - 12\cdot x$, rewrite in factored and vertex forms. Explain which form reveals the zeros, axis, extremum, and y-intercept most directly. Make a five-point table including the vertex and graph the function on the printed window.

Teacher solution and discussion

Factored: $3(x-(0))(x-(4))$. Zeros: 0, 4. Vertex: $3(x-(2))^2+(-12)$. Axis $x=2$; minimum -12. Expanded constant gives $f(0)=0$. Example table at $x=h-2, h-1, h, h+1, h+2$: outputs 0, -9, -12, -9, 0. Plot these symmetric points and a smooth parabola. Expanding either rewrite gives $3\cdot x^2 - 12\cdot x$.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T2: Use equivalent quadratic forms to explain vertices, extrema, and range (8a).

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its graph when present. Have learners propose the next step, then ask a partner to verify it using the original representation. Record more than one valid explanation.

Core task

Complete the square for $f(x) = -3x^2 + 12x$. Show the equality at each step, then explain the extremum and range for all real x . Check your rewrite by expansion and by evaluating $x=0$.

Teacher solution and discussion

Group and complete: $-3(x^2 - (4)x) + (0) = -3[(x - (2))^2 - (4)] + (0) = -3(x - (2))^2 + (12)$. The square is nonnegative. Vertex $(2, 12)$ gives maximum; range $y \leq 12$. Both forms at $x=0$ give 0. Expansion returns $-3x^2 + 12x$.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T2: Use equivalent quadratic forms to explain vertices, extrema, and range (8a).

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its graph when present. Assign reader, representer, checker and explainer roles. Each learner first makes a private prediction; compare reasoning and resolve differences with the data.

Core task

Can two quadratics have the same vertex $(1, -4)$ but different zeros? Construct two examples, rewrite each to expose its zeros, and justify your conclusion.

Teacher solution and discussion

Yes. $f = (x-1)^2 - 4 = x^2 - 2x - 3 = (x-3)(x+1)$, with zeros -1 and 3 . $g = 4(x-1)^2 - 4 = 4x^2 - 8x = 4x(x-2)$, with zeros 0 and 2 . Both have vertex $(1, -4)$, but different leading coefficients change the zero locations.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T3: Interpret exponential initial values, multipliers, and growth or decay rates (8b).

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its graph when present. Provide the printed task and neutral access supports. Preserve the first independent response; do not point to the requested answer or select a method for the learner.

Core task

$P(t)=250(26/25)^t$, where t counts years. Rewrite the base as $1+r$ or $1-r$ to expose the percentage rate. Interpret the initial value and rate, then calculate $P(1)$ and $P(3)$ exactly. Explain why the change in units is not constant.

Teacher solution and discussion

Base $26/25=1+1/25$. Initial amount 250; growth 4% per year. $P(1)=260$; $P(3)=35152/125$. The first change is 10; the next is $52/5$. Each step changes the current amount by the same proportion, so additive changes differ.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T4: Rewrite exponential expressions to explain rates over different input units (8b).

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its graph when present. Start with the learner’s prior evidence. Rebuild one representation with explicit terms and operations, then use a fresh item to check transfer without the prior cue.

Core task

A sample model $M(t)=128 \cdot (1/16)^t$ uses t in four-hour periods. Rewrite using h in hours and state the hourly decay. Verify at hour 8.

Teacher solution and discussion

$t=h/4$ and $(1/16)^{(1/4)}=1/2$, so $M=128 \cdot (1/2)^h$. Hourly decay is 50%. At $h=8$, $M=128/256=1/2$. The original uses $t=2$ and gives $128/256=1/2$ also.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T4: Rewrite exponential expressions to explain rates over different input units (8b).

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its graph when present. Ask for a concise claim supported by the representation. Collect the response before sharing a solution; use it to choose the next instructional step.

Core task

Show algebraically that $80(1.25)^t$ and $80(0.8)^{-t}$ are equivalent. Interpret the change as t increases, and explain why seeing a base below 1 in the second form does not establish decay.

Teacher solution and discussion

$0.8 = 4/5$ and $(4/5)^{-t} = (5/4)^t = 1.25^t$. Both functions grow by 25% per unit increase in t . The negative exponent reverses the effect of the base; one must rewrite to a base raised to positive t before interpreting the one-step multiplier.

Evidence and response

Look for a claim tied to specific terms, factors, operations or graph features. If a learner only names an answer, ask which values or parts of the graph support it. If a sign or operation is mishandled, reconstruct that step with a numerical analogy, then collect a fresh response. A selected option alone does not identify a unique misconception.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

Rewrite $f(x)=x^2 - 3 \cdot x - 10$ in factored form to identify its zeros. Which zero set is correct?

- A. $x = 1$ and $x = 3$
- B. $x = -10$ only
- C. $x = -5$ and $x = 2$
- D. $x = -2$ and $x = 5$

Reasoning: _____

M02 | Warm-up | T1

Rewrite $f(x)=2 \cdot x^2 - 10 \cdot x + 8$ in factored form to identify its zeros. Which zero set is correct?

- A. $x = -4$ and $x = -1$
- B. $x = 2$ and $x = 5$
- C. $x = 4$ only
- D. $x = 1$ and $x = 4$

Reasoning: _____

WARM-UP

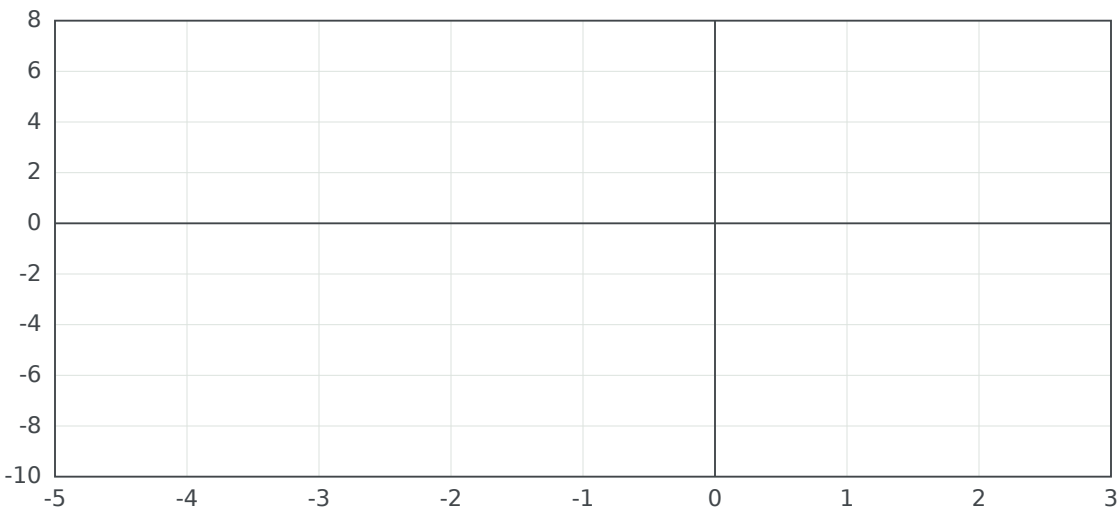
001 • Show your reasoning

T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

For $f(x)=x^2 + 2\cdot x - 8$, rewrite in factored and vertex forms. Explain which form reveals the zeros, axis, extremum, and y-intercept most directly. Make a five-point table including the vertex and graph the function on the printed window.

x	f(x)

vertical axis: _____



horizontal axis: _____

Explanation and check

WARM-UP

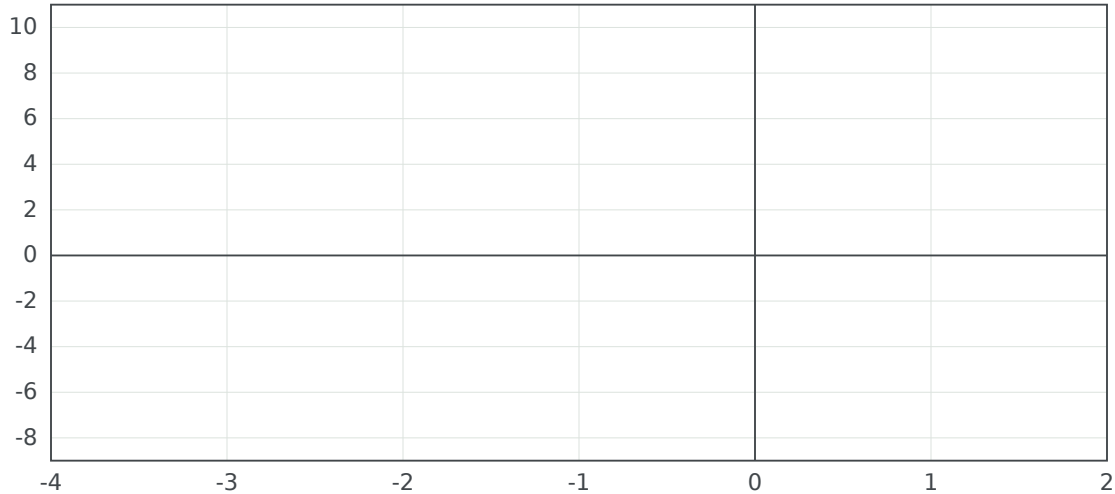
002 • Show your reasoning

T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

For $f(x)=2\cdot x^2 + 4\cdot x - 6$, rewrite in factored and vertex forms. Explain which form reveals the zeros, axis, extremum, and y-intercept most directly. Make a five-point table including the vertex and graph the function on the printed window.

x	f(x)

vertical axis: _____



horizontal axis: _____

Explanation and check

WORKED TEACHER KEYS

M01 / M02**M01 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).****Answer: D. $x = -2$ and $x = 5$**

$f(x)=1(x-(-2))(x-(5))$. The product is zero exactly when $x=-2$ or $x=5$. Expanding returns $x^2 - 3x - 10$.

Check each alternative

- A. These use a coefficient and sum of roots, not the zero-product rule.
- B. The product of roots does not by itself give the zeros.
- C. These reverse the signs of the zeros implied by the factors.
- D. Correct: $1(x-(-2))(x-(5))=0$ gives these zeros.

M02 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).**Answer: D. $x = 1$ and $x = 4$**

$f(x)=2(x-(1))(x-(4))$. The product is zero exactly when $x=1$ or $x=4$. Expanding returns $2x^2 - 10x + 8$.

Check each alternative

- A. These reverse the signs of the zeros implied by the factors.
- B. These use a coefficient and sum of roots, not the zero-product rule.
- C. The product of roots does not by itself give the zeros.
- D. Correct: $2(x-(1))(x-(4))=0$ gives these zeros.

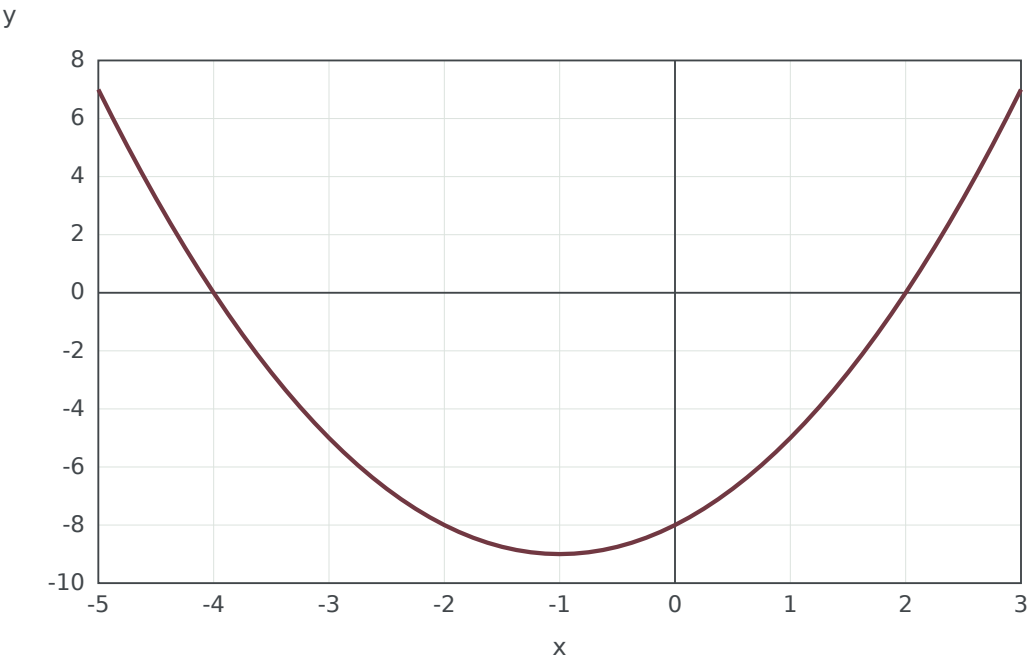
WORKED TEACHER KEYS

001

001 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

Factored: $1(x - (-4))(x - (2))$. Zeros: -4, 2. Vertex: $1(x - (-1))^2 + (-9)$. Axis $x = -1$; minimum -9. Expanded constant gives $f(0) = -8$. Example table at $x = h - 2, h - 1, h, h + 1, h + 2$: outputs -5, -8, -9, -8, -5. Plot these symmetric points and a smooth parabola. Expanding either rewrite gives $x^2 + 2 \cdot x - 8$.

x	f(x)
-3	-5
-2	-8
-1	-9
0	-8
1	-5



Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.

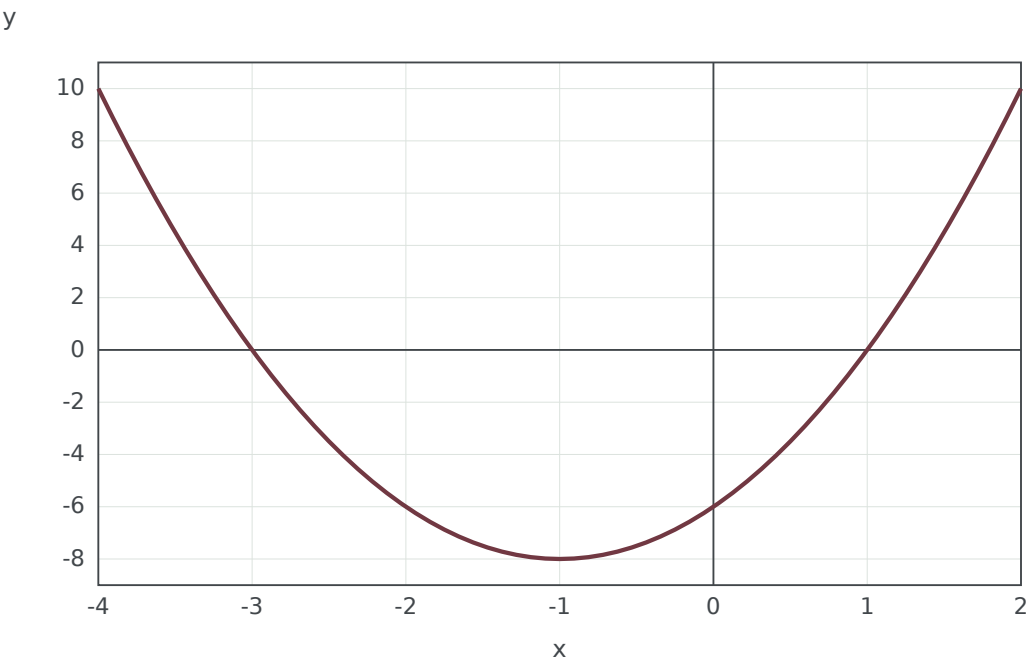
WORKED TEACHER KEYS

002

002 • T1: Rewrite quadratics to reveal and explain zeros and intercepts (8a).

Factored: $2(x - (-3))(x - (1))$. Zeros: -3, 1. Vertex: $2(x - (-1))^2 + (-8)$. Axis $x = -1$; minimum -8. Expanded constant gives $f(0) = -6$. Example table at $x = h - 2, h - 1, h, h + 1, h + 2$: outputs 0, -6, -8, -6, 0. Plot these symmetric points and a smooth parabola. Expanding either rewrite gives $2 \cdot x^2 + 4 \cdot x - 6$.

x	f(x)
-3	0
-2	-6
-1	-8
0	-6
1	0



Evidence to assess

Assess accurate representation, a valid justification, and a conclusion consistent with the domain and units. Accept equivalent reasoning or another construction meeting every condition. An answer alone does not establish a unique misconception.