

TEACHER AND STUDENT RESOURCE

Test population models with simulation



NC Math 2 • NC.M2.S-IC.2

Making Inference and Justifying Conclusions

Seven instructional sections with 30 multiple-choice and 30 open or performance tasks, teaching scripts, original mathematical visuals, generous workspaces, and worked keys.

Free sample | Contents

NC.M2.S-IC.2 | NC Math 2 mathematics

This sample contains 22 complete selected pages from the 124-page resource, plus this sample contents page. Original printed page numbers are retained. Gaps are intentional.

Included original pages: 1-16, 32-33, 62-63, 92-93

Representative activities

M01: sample PDF page(s) 17; original page(s) 16

M02: sample PDF page(s) 17; original page(s) 16

O01: sample PDF page(s) 18; original page(s) 32

O02: sample PDF page(s) 19; original page(s) 33

Selected teacher prompts and keys: original page(s) 62-63, 92-93

Using this sample

Read the preparation and access guidance before using a selected activity.
Keep teacher keys and private preparation pages separate from student copies.
Use the original resource contents to see the scope of the complete packet.

Complete paid download

The complete resource includes the full PDF task bank and teaching materials.

ALIGNMENT

Standard and learning targets

Use simulation to determine whether the experimental probability generated by sample data is consistent with the theoretical probability based on known information about the population.

T1 Encode known population probabilities and match the actual sampling process in a repeatable simulation.

T2 Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

T3 Use simulated variation to judge consistency with theoretical probability, while explaining uncertainty and limits of the conclusion.

Each item names its own target. The bank supports multiple sessions; do not assign all 60 tasks in one sitting.

NAVIGATION

Choose evidence deliberately

Warm-up: M01-M04 and O01-O04

I Do: M05-M08 and O05-O08

We Do: M09-M13 and O09-O13

Collaborative practice: M14-M18 and O14-O18

Independent practice: M19-M23 and O19-O23

Small-group reteach: M24-M27 and O24-O27

Exit ticket: M28-M30 and O28-O30

T1: M01, M02, M03, M09, M10, M13, M15, M16, M20, M23, M25, M28, O02, O03, O07, O10, O14, O15, O18, O20, O24, O28

T2: M04, M07, M08, M11, M14, M19, M21, M24, M27, O01, O04, O06, O08, O12, O19, O23, O26

T3: M05, M06, M12, M17, M18, M22, M26, M29, M30, O05, O09, O11, O13, O16, O17, O21, O22, O25, O27, O29, O30

DAILY LEARNING TARGETS

A multi-session teaching path

Session 1 Identify and represent

Learning target: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Learning target: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

Use Warm-up, I Do, and early We Do tasks. Ask for a first response before showing the model.

Session 2 Explain and connect

Learning target: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

Learning target: Use simulated variation to judge consistency with theoretical probability, while explaining uncertainty and limits of the conclusion.

Use We Do and collaborative tasks. Compare approaches and justify each step with the original conditions.

Session 3 Transfer independently

Learning target: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Learning target: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

Learning target: Use simulated variation to judge consistency with theoretical probability, while explaining uncertainty and limits of the conclusion.

Select independent items spanning the targets, reteach an evidenced gap, and use an unrehearsed exit item.

PLANNING

Materials, access and pacing

Materials

Pencils, erasers, straightedge, student pages, and optional graph paper. A calculator may support arithmetic after a learner states the intended relationship. No device or paid software is required.

Prerequisites

Compute fractions, relative frequencies and complements. Distinguish a population model from an observed sample. Read frequency tables and bar charts; recognize how replacement and independence affect a process.

Pacing

Use a short diagnostic warm-up, a model and shared task in one session; collaborative and independent reasoning in later sessions; then targeted reteaching and an exit task. Select two to four tasks at a time based on evidence. Each section's full bank is reusable, not mandatory in one lesson.

Neutral access

Read directions aloud without interpreting the graph, enlarge a page, supply a writing or pointing response method, and allow a learner to dictate an explanation. Keep axes, units, data and mathematical demand unchanged.

Separate instruction from evidence

Keep the learner's first response. Record answer-supplying cues separately from access supports. A completed teacher graph or worked key is for discussion after independent work, never a student construction template.

Review convention

All datasets and supplied simulation outputs are original instructional examples, not claims about real populations. Each question specifies its population probability and sampling model. A 0.05 guide, where stated, is a predeclared classroom convention, not proof or a universal definition of inconsistency. Include equality in at-least-as-extreme comparisons. Independently generated digits 0–9 or pairs 00–99 are uniform unless stated otherwise. Rejected random values do not consume trials.

TEACHER MODEL

Compare a sample with model based variation

A known population is 30% red. An actual sample of 20 independent replacement draws has 12 reds. A matched simulation produces the frequencies below for 1000 samples. Does this high result look consistent under a predeclared 0.05 unusual-result guide?

Red count	Simulated samples
0-5	410
6-9	560
10-11	25
12-20	5
Total	1000

Each simulated sample contains 20 independent draws using $p(\text{red})=0.30$.

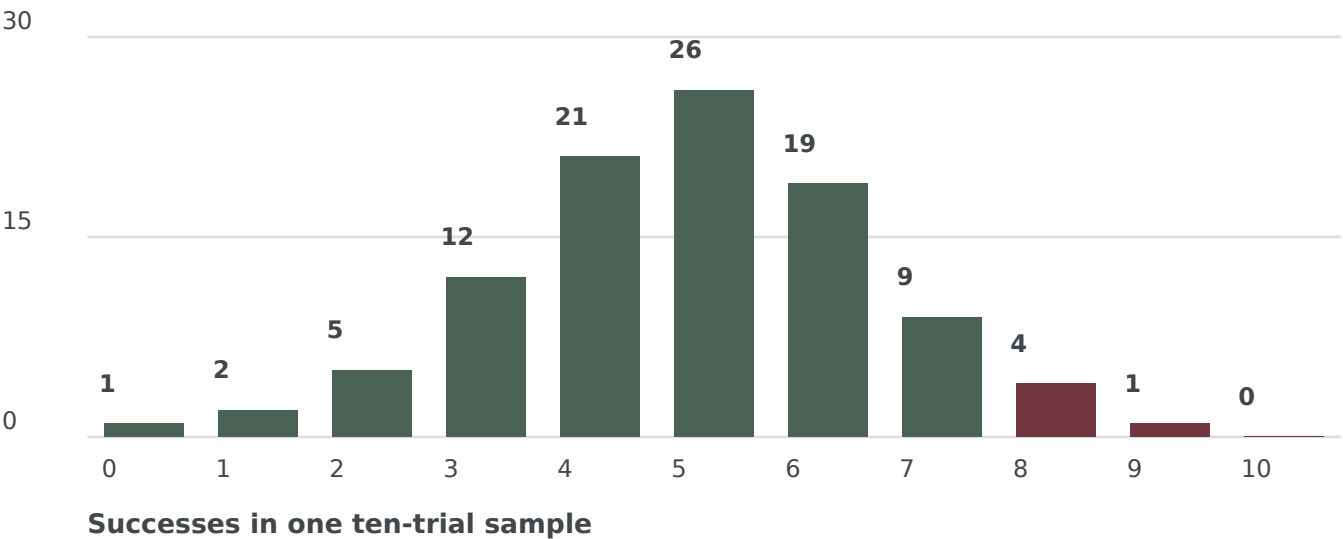
The actual experimental rate is $12/20=0.60$; the known theoretical rate is 0.30. The upper-tail estimate is $5/1000=0.005$, below 0.05. This high result is unusual under the specified model, so check the population assumptions and sampling process. The 0.005 is a frequency of whole simulated samples, not a probability that the model is true.

VISUAL ANCHOR

Keep the two probability levels separate

For independent fair trials, the theoretical single-trial success probability is 0.50. The chart shows variation in success counts across 100 samples of 10 trials.

Number of simulated samples



Highlighted bars show counts at least 8; all 100 samples form the denominator.

An actual sample with 8 successes has experimental rate 0.80. The highlighted samples give an upper-tail estimate $(4+1+0)/100=0.05$. The 0.80 describes the observed sample, 0.50 describes one model trial, and 0.05 describes entire samples at least as high. These three probabilities answer different questions.

REUSABLE REASONING GUIDE

Represent, justify, interpret

Name: _____ Date: _____

Sample space, events and known information**Representation and calculation****Justification or check****Conclusion in context**

WARM-UP

Teaching script

Core: 001 • T2: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

Optional bank: M01–M04; O01–O04. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O01, including its visual when present. Keep the population probability visible while learners label one digit, one sample and one repetition. Have learners predict variation before calculating the supplied sample records.

Core task

A known population is 30% red. Model independent draws with replacement by digits 0–2=red and 3–9=blue. Four five-draw samples are 02749, 98163, 20411 and 73802. Construct a record table with each sample's red count and red proportion. An actual sample has 4 reds in 5 draws. Estimate the fraction of simulated samples at least as high and state a limitation.

Teacher solution and discussion

The four counts are 2, 1, 4, 2, giving proportions 0.40, 0.20, 0.80, 0.40. The actual proportion is $4/5=0.80$. Only sample 20411 has at least 4 reds, so the estimated upper-tail rate is $1/4=0.25$. Four repetitions are too few for a stable rarity judgment; retain the $p=0.30$ model and run many more matched samples.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M01 to compare all alternatives after evidence is collected.

I DO

Teaching script

Core: 005 • T3: Use simulated variation to judge consistency with theoretical probability, while explaining uncertainty and limits of the conclusion.

Optional bank: M05–M08; O05–O08. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–12 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O05, including its visual when present. Model a full comparison aloud: state the known probability, preserve sample size and independence, locate the observed statistic, include equally extreme results, then qualify the conclusion.

Core task

A known population has success probability 0.25 for independent replacement draws. An actual eight-draw sample has five successes. Use the supplied 100-sample simulation chart to estimate the at-least-as-high rate. Under a predeclared guide of at most 0.05, write a qualified consistency conclusion and distinguish the single-trial probability from the across-sample tail probability.

Teacher solution and discussion

The observed experimental success probability is $5/8=0.625$, compared with the known theoretical 0.25. The simulated upper tail includes counts 5–8: $2+1+0+0=3$ of 100, or 0.03. That is at most 0.05, so this high sample result is unusual under the stated model and warrants checking the assumptions or process. The 0.03 tail rate concerns entire samples; it is not the 0.25 chance of one success.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M05 to compare all alternatives after evidence is collected.

WE DO

Teaching script

Core: 009 • T3: Use simulated variation to judge consistency with theoretical probability, while explaining uncertainty and limits of the conclusion.

Optional bank: M09–M13; O09–O13. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O09, including its visual when present. Assign the class a debugging role. Test whether the counter resets, the sample length matches and the comparison uses the same statistic before trusting any simulation output.

Core task

A fair binary model is tested with samples of five independent trials. An actual sample has 4 successes. A 100-sample simulation has frequencies 4, 15, 31, 30, 16, 4 for counts 0, 1, 2, 3, 4, 5. A learner uses $16/100$ because exactly 4 matches were observed. Correct the comparison for an unusually high rate, then interpret it under a predeclared 0.05 guide.

Teacher solution and discussion

The actual success rate is $4/5=0.80$; theory is 0.50. A high-tail comparison includes counts 4 and 5, totaling $16+4=20$ repetitions, so the estimate is 0.20. The exact-match rate 0.16 omits even higher results. Because $0.20>0.05$, this check does not strongly contradict the fair model, although it does not prove fairness.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M09 to compare all alternatives after evidence is collected.

COLLABORATIVE PRACTICE

Teaching script

Core: O14 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Optional bank: M14–M18; O14–O18. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O14, including its visual when present. Assign model designer, random-stream reader, recorder and skeptic roles. Predeclare the comparison before generating outcomes; exchange records and audit accepted values and sample boundaries.

Core task

A production model says each independent item has a 0.15 chance of a stripe. A sample of 30 contains 10 striped items. Design a reproducible random-digit-pair simulation to assess an unusually high striped proportion. Specify mapping, repetition length, number of repetitions, statistic, comparison and a decision guide chosen in advance. Do not replace 0.15 with 10/30.

Teacher solution and discussion

Use independent uniform pairs 00–99: 00–14 striped, 15–99 unstriped. In each repetition generate 30 pairs, count stripes and record count/30. Reset the counter and repeat, for example, 2000 times. The actual proportion is $10/30 = 1/3$. Estimate the upper-tail rate by repetitions with count at least 10 divided by 2000. Predeclare, for example, that rates at most 0.05 flag an unusual result, while retaining simulation uncertainty and avoiding proof claims.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M14 to compare all alternatives after evidence is collected.

INDEPENDENT PRACTICE

Teaching script

Core: O19 • T2: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

Optional bank: M19–M23; O19–O23. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 15–20 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O19, including its visual when present. Require learners to connect the simulation to the actual data-generating process, not just calculate a percentage. Collect a complete written conclusion and the assumption it depends on.

Core task

A known $p=0.10$ process is checked with 20 independent trials. An actual sample has 5 successes. A simulation summary for 1000 samples is: counts 0–2: 680; count 3: 190; count 4: 85; count 5: 32; counts 6–20: 13. Construct a table of frequencies and relative frequencies. Estimate the high-tail rate and interpret it under a predeclared 0.05 guide.

Teacher solution and discussion

The relative frequencies are 0.680, 0.190, 0.085, 0.032, 0.013 and total 1. The observed rate is $5/20=0.25$; the theoretical rate is 0.10. At least 5 includes $32+13=45$ samples, giving 0.045. Under the stated guide this is unusual, though near the guide and subject to simulation uncertainty; it is not proof that the model is false.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M19 to compare all alternatives after evidence is collected.

SMALL-GROUP RETEACH

Teaching script

Core: O24 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Optional bank: M24–M27; O24–O27. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 10–15 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O24, including its visual when present. Use three nested labels: individual trial, one complete sample, many repeated samples. Contrast an expected count with a fixed count and a zero simulated estimate with a logically impossible event.

Core task

A $p=0.40$ simulation maps digits 0–3 to success. A learner stops each sample as soon as four successes occur, but the actual data came from a fixed sample of 10 trials. Explain the mismatch and rewrite the instructions. Give the correct statistic and a suitable comparison for an actual count of 7 successes.

Teacher solution and discussion

Stopping at the fourth success gives variable sample lengths and forces the success count to 4, unlike the fixed 10-trial design. Instead, generate exactly 10 independent digits per repetition, count successes, record count/10 and reset for the next repetition. The actual proportion is $7/10=0.70$. For an unusually high question, use the fraction of repeated ten-trial samples with count at least 7.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M24 to compare all alternatives after evidence is collected.

EXIT TICKET

Teaching script

Core: O28 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Optional bank: M28–M30; O28–O30. Select tasks across more than one meeting if needed.

Suggested discussion/work time: 5–8 minutes. Adjust for access and evidence; this is not a speed criterion.

Display student page O28, including its visual when present. Ask for an unprompted model, statistic, simulation comparison and limited conclusion. Use omitted model assumptions or confused denominators to choose the next lesson.

Core task

A toy model has 25 orange and 75 purple tokens. An actual sample consists of 12 independent uniform draws with replacement and has 7 oranges. Write a complete simulation plan and the experimental and theoretical probabilities. State exactly which simulated samples count as at least as high; include a way another person could reproduce and audit your procedure.

Teacher solution and discussion

Theory gives $25/100=0.25$; observation gives $7/12$. Use independent uniform pairs 00–99, with 00–24 orange and 25–99 purple. Generate 12 accepted pairs per sample, record orange count/12, reset, and repeat many times, for example 5000. Count samples with at least 7 oranges and divide by 5000. Save the digit stream or random-generator seed, mapping, sample boundaries and counts so another person can reproduce and verify the run.

Evidence and response

Look for the population model, a matching sample-generating procedure, the correct sample statistic, a simulation-based comparison and a qualified contextual conclusion. A raw difference between experimental and theoretical probabilities is not enough. Distinguish individual trials, repeated samples and uncertainty in an estimated tail rate.

Closure: Ask learners to state one feature they verified and one check they used. Use the worked key for M28 to compare all alternatives after evidence is collected.

MULTIPLE-CHOICE BANK

M01 / M02 • Select and justify**M01 | Warm-up | T1**

A population contains 30 red and 70 blue counters. Uniform draws with replacement are independently mixed. A sample of 20 draws contains 9 reds. Which pair gives theoretical and experimental red probabilities, respectively?

- A. 0.30 and 0.09
- B. 0.30 and 0.45
- C. 0.45 and 0.30
- D. 0.70 and 0.55

Reasoning: _____

M02 | Warm-up | T1

Which rule uses independent uniform digits 0-9 to simulate success probability 0.30?

- A. Digits 1-3 are success; discard every other digit.
- B. Digits 0, 1, 2 are success; 3-9 are failure.
- C. Digits 0-3 are success; 4-9 are failure.
- D. Digit 3 is success; all other digits are failure.

Reasoning: _____

WARM-UP

001 • Show your reasoning

T2: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

A known population is 30% red. Model independent draws with replacement by digits 0-2=red and 3-9=blue. Four five-draw samples are 02749, 98163, 20411 and 73802. Construct a record table with each sample’s red count and red proportion. An actual sample has 4 reds in 5 draws. Estimate the fraction of simulated samples at least as high and state a limitation.

Representation / calculation

Explanation / check

WARM-UP

002 • Show your reasoning

T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Design a fair-coin simulation using whether a uniform random digit is even or odd. Apply it to four samples of four digits: 1863, 0072, 9541, 6208. Make a completed heads-count record table. An actual four-toss sample is all heads. Estimate the high-tail rate from these repetitions, and explain why this does not settle fairness.

Representation / calculation

Explanation / check

WORKED TEACHER KEYS

M01

M01 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Answer: B. 0.30 and 0.45

The known population gives $30/100=0.30$. The observed sample gives $9/20=0.45$. Their difference alone does not establish inconsistency.

Check each alternative

- A. 9 must be divided by the sample size 20, not the population size 100.
- B. The known population gives $30/100=0.30$. The observed sample gives $9/20=0.45$. Their difference alone does not establish inconsistency.
- C. This reverses population probability and sample proportion.
- D. These are blue rather than red probabilities.

Evidence to assess

Require both denominators and the population/sample distinction.

WORKED TEACHER KEYS

M02

M02 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Answer: B. Digits 0, 1, 2 are success; 3-9 are failure.

Exactly 3 of 10 equally likely digits must count as successes, and every digit is one trial.

Check each alternative

- A. Discarding every failure leaves only successes among accepted trials, not a 0.30 process.
- B. Exactly 3 of 10 equally likely digits must count as successes, and every digit is one trial.
- C. Four successful digits would simulate 0.40.
- D. One successful digit would simulate 0.10.

Evidence to assess

Require an exhaustive mapping with the intended probability.

WORKED TEACHER KEYS

001

001 • T2: Generate or summarize repeated samples, calculate experimental probabilities and compare matched statistics using an explicit extreme-result rule.

The four counts are 2, 1, 4, 2, giving proportions 0.40, 0.20, 0.80, 0.40. The actual proportion is $\frac{4}{5}=0.80$. Only sample 20411 has at least 4 reds, so the estimated upper-tail rate is $\frac{1}{4}=0.25$. Four repetitions are too few for a stable rarity judgment; retain the $p=0.30$ model and run many more matched samples.

Digit sample	Red count	Red proportion
02749	2	$\frac{2}{5}=0.40$
98163	1	$\frac{1}{5}=0.20$
20411	4	$\frac{4}{5}=0.80$
73802	2	$\frac{2}{5}=0.40$

Each row is one simulated sample of five draws.

Evidence to assess

Item evidence: four correct counts; four proportions; tail estimate $\frac{1}{4}$; small-run limitation. Award one point per named requirement, up to 4; accept justified alternative designs and appropriately qualified conclusions.

WORKED TEACHER KEYS

002

002 • T1: Encode known population probabilities and match the actual sampling process in a repeatable simulation.

Let 0, 2, 4, 6, 8 mean heads and odd digits mean tails. Each group has five digits, so the theoretical heads probability is 1/2. Counts are 2, 3, 1, 4, with proportions 1/2, 3/4, 1/4, 1. One of four repetitions is all heads, estimating the upper-tail rate as 1/4. A four-repetition estimate is very unstable and cannot establish or refute fairness by itself.

Digits	Heads count	Heads proportion
1863	2	1/2
0072	3	3/4
9541	1	1/4
6208	4	1

Even-digit heads mapping; each row contains four independent trials.

Evidence to assess

Item evidence: equal-probability mapping; complete four-row record; upper-tail 1/4; limited inference. Award one point per named requirement, up to 4; accept justified alternative designs and appropriately qualified conclusions.